Name: _____

*Date:*_____

Locker Problems

1 - Problem

A special chamber has a grid of 100 lockers, numbered in order from 1 - 100. Initially, **all lockers are *closed*.**

We have 100 students in a line, each given a number from 1 - 100. Each student will pass through the lockers, opening and closing lockers following this pattern:

- Student #1 changes every single locker to *open*
- Student #2 changes every 2nd locker to *closed* (i.e. Lockers 2, 4, 6, 8, etc.)
- Student #3 changes the state of every 3rd locker. If the locker is *open*, they will *close* it. If the locker is *closed*, they will *open* it.

  …
- Student #100 changes the state of every 100th locker (i.e. just the last locker)

Question: After all 100 students have taken their turn, **which lockers will end up *open***?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Use this space to write down your thoughts! What questions come up along the way?

Optional hint questions:

Pretend that you are locker X. Which students would you need to "worry" about changing your state? How many of these students are there?

Will 1 be open? Will 2 be open? What about 3 or 4? Pattern?

2 - More Questions

Thinking about the above problem:

1) Which lockers get touched (changed) only twice?

2) Which locker(s) get touched by both: Student #20 and Student #3? What about Student #6 and Student #15?

We can also try changing the rules:

3) What if the students went shuffled up, in a different order - would that change which lockers end up open?

4) What if Student #3 went twice (took an extra turn)?

5) What if Student #3 was sick (didn't go)? What about if both Student #3 and #9 were sick?

6) Can we find which students to send, to get only the *prime number* lockers to end up open?

7) Can we find which students to send, so that only locker #1 ends up open (the rest are closed)?

3 - Demo

Let's try to run through the locker process! Instead of 100, we will use a smaller number, 25.

Also, to represent lockers, we can use items that can be flipped, such as: paper cups, 2-colored tokens, playing cards, etc. Arrange these in a numbered grid.

Decide which side is "open" and which side is "closed"; for instance:

- Cup face up = Open locker
- Cup face down = Closed locker

25 students will then run through and flip the items according to the pattern!

Appendix

Material adapted from:

- (Elgin Johnston) Lockers: An Open-and-Shut Case
- (Linda Green) Variations on the Locker Problem
- (Lisa Winer) TED ED: Can you solve the locker riddle? - Lisa Winer

More to check out:

- Even/odd
- Relatively prime stars
- Euclid's game
- Sieve of Eratosthenes
- How many numbers are relatively prime to a given number?
- What is the smallest number with exactly 7 factors?