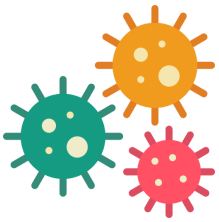
The Pool Testing Lab

At Chapel Hill Math Circle - Elementary

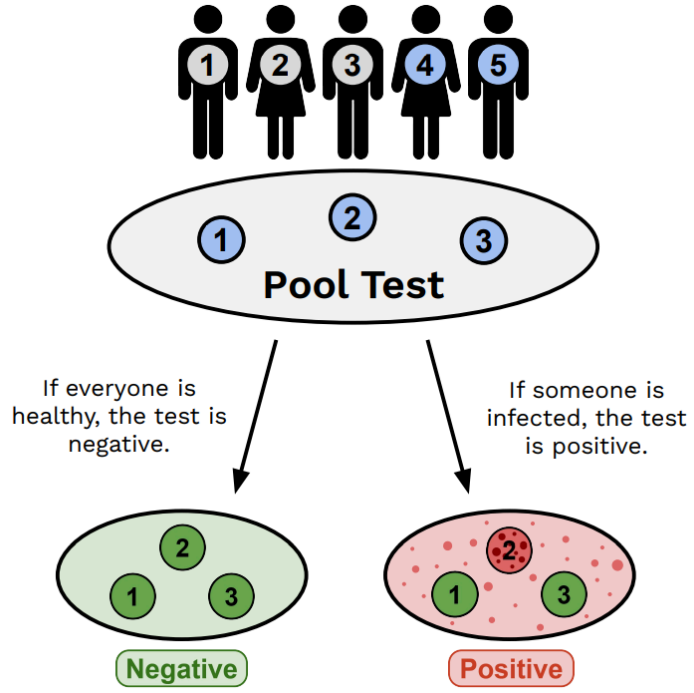


Imagine: A new viral infection called "PicklePox" has hit the UNC campus. We have a group of students, and the Lab needs to figure out who is sick. How can we do this, without testing each student individually?

One technique we can use is called **pool testing**. This lets you test a set of students all at once.

For example:
Say you have 5 students and you know that only 1 is sick. You can pick a pool of 3 students to test all at once. For this pool, there are 2 possible test results:

- **Positive (+)**: At least one student in the pool is sick
- **Negative (-)**: No virus. Everyone in the pool is healthy



Warm Up:

1) A group of 2 students arrive in the lab, and 1 of them is infected. The Lab Director asks you if you can find the PicklePox student using only 1 pool test. Is this possible?

2) While you aren't looking, a new group of 3 different students comes into the lab. Again, only 1 of them is sick. Will you still be able to find the PicklePox student with 1 test?

    a) Would it be possible with more pool tests? Try to come up with the minimum number of tests you need to guarantee you'll find the sick student.*

**Remember that we can "get lucky" and find the infected student in one test, but we want to find a strategy that will guarantee we find the infected student (no matter how lucky or unlucky we are).*

Your Lab Notebook

Task 1: Sequential Testing

Goal: Find the infected student using as few pool tests as possible.

Rules

- In each group, exactly 1 person is infected.
- You can do pool tests of any size (including a pool of just one person).
- You will usually need to do more than one pool test to find the infected person. You can test the same person any number of times.

1) There are 6 students and exactly 1 student is infected. What is the *fewest* number of tests you need to guarantee that you will always be able to find the infected student?

2) What if there are 7 students? 8 students? 9?

Try out some other size groups. Record your findings in this chart below:

	1 infected student
Number of students	Minimum # of Tests
3 4 5 6 7 8 9	

3) If you are only able to do 5 tests, what is the largest number of students you can test so that you can always find the infected one?

Task 2: Parallel Testing

One day, you only have enough time to do one round of pool testing. You can do as many pool tests as you want, but you must do all of the tests at once. You cannot use the result of one pool test to decide how you'll do another pool test.

1) If there are 6 students and exactly 1 student is infected, what is the fewest number of pool tests you need to guarantee that you will always be able to find the infected person?

2) What if there are 7 students? 8 students? 9?

3) If you are only able to do 5 tests, what is the largest number of students you can test so that you can always find the infected one?

4) The clumsy lab technician often spills one of the tests. However, you don't know which test result is going to be lost. Knowing this, how should you do your pool testing to guarantee that you'll find the infected person?

Task 3: Multiple infected students

Pickpox infections have become more frequent at UNC. Now we are looking at groups of students with more than 1 sick student, and we want to find all of the sick ones. (You don't have to run all the tests at once anymore)

Start with 2 infected students:

1) There are 6 students and exactly 2 students are infected. What is the fewest number of tests you need to guarantee that you will always be able to find both of the infected students?

2) What if there are 7 students? 8 students? 9?

3) If you are only able to do 5 tests, what is the largest number of students you can test so that you can always find both of the infected ones?

Now keep going and try larger numbers of infected students:

4) What if exactly 3 students are infected? 4 students ? 5? More?

	2 infected students	3 infected students	More?
Number of students	Minimum # of Tests	Minimum # of Tests	Minimum # of Tests
3			
4			
5			
6			
7			
8			
9			

5) What if there is an unknown number of students infected?

6) Is pool testing *always* more efficient (meaning it requires fewer tests) than testing each student one-by-one?

Group - Act it out

Let's have 6 students volunteer from the class to be "actors".

The instructor will secretly pick 2 of them to be "sick" with PicklePox and will not tell anyone but the actors.

The "actors" will stand in front of the class. The rest of the class will then try to figure out which "actors" are sick by running select pools they want to test. The instructor tells the result of the test (positive or negative) and keeps a tally of the number of tests used on the board.

1) Keep any notes you wish to take here.

2) Which 2 "actors" did you find were sick with PicklePox?

Sources

- Problems and content thanks to JRMF (https://jrmf.org/puzzle/pool-testing/)
- https://www.ams.org/publicoutreach/feature-column/fc-2020-10
- Virus Image from: https://cdn-icons-png.flaticon.com/512/2904/2904332.png