Chapel Hill Math Circle
Session 8 – February 14, 2026: Counting Salad Bar Choices
Beginners' Group (grades 1-3), 10:30-11:30a
Mr. Barman – dilip@trianglemathinstitute.com

Supplies needed: colored pencils and scissors

I hope that you have a happy Valentine's Day. Do you like eating salads? If you are free later today, watch my Valentine's Day cooking show (I'm making a chocolate dessert) in my monthly plant-based show somanycooks.com. Let's look at salad bars!

Do you like salad bars?

I love salad bars – do you? If you don't know what a salad bar is, it's a place where items like tomatoes, olives, lettuce, cabbage, tofu, and more are available; you can select items that you want[1].

Locally we have very nice salad bars at Weaver Street Market and Whole Foods Market, as well as several other places. Let's think about some initial questions.

How do you make choices at a salad bar? (If you've never eaten at a salad bar, imagine that you are at a restaurant where you serve yourself and have over 20 choices. Or maybe your family lays out taco ingredients and you can select items like lettuce, kale, black beans, 2 kinds of salsa, and more.) What would your choices be like? Would you go back again and again and get a few things? Would you pile on lots of choices like peas and carrots and tangerine slices and beans and more? What does "choice" mean – getting an ingredient? Getting different amounts of ingredients? Take a few minutes and imagine what this might be like.

Notes on how you like to choose from a salad bar

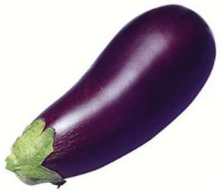
[1] The closeup picture below is from flickr.com/photos/dinesarasota/6797105991, accessed Oct. 10, 2023; it's a photograph by Larry Hoffman, "Design Your Own Salad Bar", taken on Jan. 29, 2012 and shareable with attribution. The other picture is one I took of part of the Weaver Street Market salad bar on February 16, 2024.

Salad bars offer many choices

- Let's say that you have 1 item in the salad bar, perhaps apples. You have two choices of salads – apples or nothing.
- For completeness, you could make 0 salads if there are 0 items from which to choose.
- How many different salads could you make with 2 choices, apples and beans?
- How many different salads could you make with 3 choices, apples, beans, and carrots?
- How about with 4 choices, including dragonfruit?
- How about with 5 choices, including eggplant?

Let's try it. Please complete this chart. You can use the salad bar ingredient images on the last page to cut out and build each of these salads.

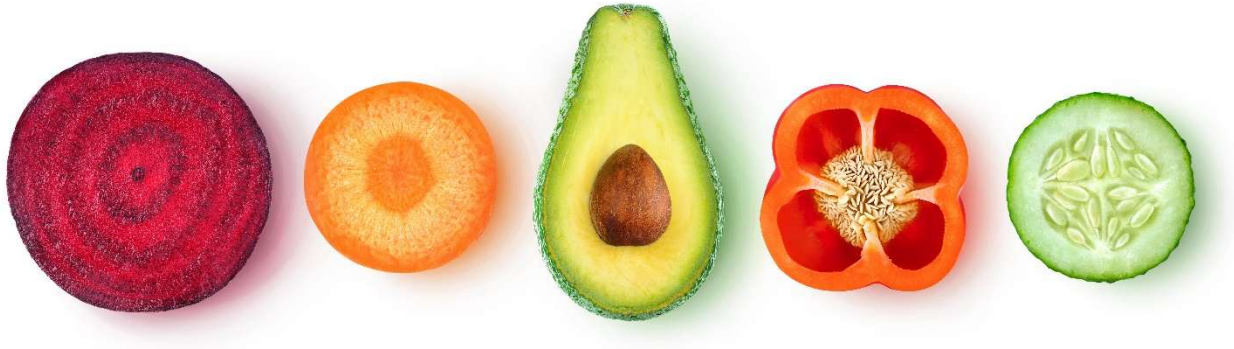
# Ingredients	Ingredients	# Choices	Choices
0	none	1	Nothing
1	A	2	Nothing or A
2			
3			
4			
5			



So what is the pattern?

Did you find that the number of choices doubles each time? If you can make over 1000 (actually 1024) different salads with 10 ingredients, how many salad bar choices would you have if you have 11 ingredients?

Please take a few minutes to think about these questions. **Do the choices keep doubling? Why or why not? About how many choices would you have if you had 11 ingredients** given that you can make about 1000 salads with 10 ingredients?



So how many salads could you make?

Your exploration was what to do with a salad bar with 3 ingredients (apples, beans, and carrots) or with 4 ingredients (adding dragonfruit). Then you were to share how many salads could be made with 11 ingredients if there are over 1000 (actually, 1024) salads that 10 ingredients make.

With **0 ingredients**, there is only 1 salad you could make – an empty plate.

With **1 ingredient**, say, apples, you have a choice of an empty plate or a plate of apples, so there are 2 possible salads.

With **2 ingredients**, say A and B, you could have nothing, a plate of A, a plate of B, or a plate of both, so 4 possibilities.

With **3 ingredients**, say A, B, and C, you could have nothing, just A, just B, just C, A and B, A and C, B and C, or all 3 – 8 choices.

With **4 ingredients**, A, B, C, and D, you could have nothing; a 1-ingredient plate (4 ways – just A, just B, just C, or just D), a 2-ingredient plate (AB, AC, AD, BC, BD, CD – 6 ways), a 3-ingredient plate (we can readily calculate this as 4 choices by considering not the 3 ingredients that are "in" but the one that is "out" – i.e., all but A, all but B, all but C, or all but D), or all 4 ingredients – 1 + 4 + 6 + 4 + 1 = 16 ways.

Yikes, there seems to be a pattern here! We will come back to this (1 + 4 + 6 + 4 + 1) but let's consider the case of 5 ingredients to see the pattern evolving. **Can you summarize again in this way the number of plates that can be made with 5 ingredients?** Look at your chart if you wish.

With **5 ingredients**, A through E, you could:

- Have an empty plate (1 way)
- Pick any 1 ingredient (5 ways)
- Pick 2 ingredients: AB, AC, AD, AE; BC, BD, BE; CD, CE; or DE (4 + 3 + 2 + 1 = 10 ways)
- Pick 3 ingredients – but isn't this the same as not picking two ingredients? (10 ways)
- Pick 4 ingredients – that is the same as picking one ingredient not to include (5 ways)
- Pick all 5 ingredients (1 way)

So we have 1 + 5 + 10 + 10 + 5 + 1 = twice 1 + 5 + 10 = 32.

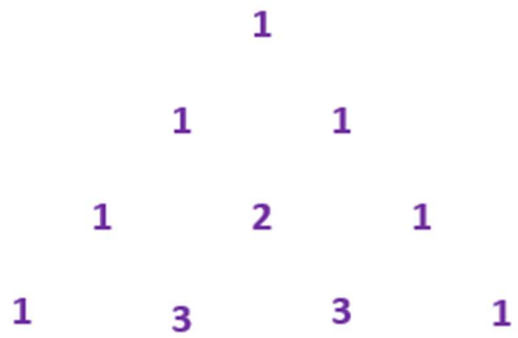
Let's summarize what we have so far:

- 0 ingredients: 1 way
- 1 ingredient: 1 + 1 = 2 ways
- 2 ingredients: 1 + 2 + 1 = 4 ways
- 3 ingredients: 1 + 3 + 3 + 1 = 8 ways
- 4 ingredients: 1 + 4 + 6 + 4 + 1 = 16 ways
- 5 ingredients: 1 + 5 + 10 + 10 + 5 + 1 = 32 ways

What patterns do you see? Do you remember Pascal's Triangle (if you were here in the fall)?

Pascal's Triangle

Write down the number 1 near the horizontal center of a piece of paper. On a line below it (the next row), start the row with a 1 and end it with a 1 on either side of the top 1. For the following row, start and end with a 1 but then for the middle value add the two numbers. Keep going; start and end each row with a 1 and find out the other values by adding. You should end up with a triangle that looks like the following after 4 rows:



Blaise Pascal

This triangle was popularized in 1653 by Blaise Pascal[2], a remarkable mathematician and philosopher who was brilliant at math as a child. He created one of the first calculators and helped to create important areas of math, economics, and social science.
He didn't invent it; it was well known starting over 1000 years earlier in India, Greece, China, and Persia. He knew this and didn't take credit but another mathematician, Raymond de Montmort, named it after Pascal in 1708.

[2] The picture here is from an engraving of Pascal by Henry Hoppner Meyer almost 200 years ago in 1833; I got it from britannica.com/biography/Blaise-Pascal, accessed Jan. 9, 2023.

Make a Pascal's Triangle

Make a Pascal's Triangle to at least 5-7 rows. What patterns do you find? How does this relate to the Salad Bar Problem?

Activities for you to consider

Counting ways to make something like a salad or a bowl of vegan ice cream with different flabors is an example of counting when order doesn't matter. Did you get a plate with apples and carrots? And your friend got a plate with bananas, carrots, and dragonfruit?

In the future we will look at problems where order matters. One example is building an ice cream cone. Getting vanilla on the bottom and chocolate on top of a 2-scoop cone is different than starting with chocolate and then having vanilla.

- What does Pascal's Triangle have to do with the Salad Bar Problem?
- There are about 330 million people in the United States. How many choices does a salad bar have to have to make sure that every single person in the country could get a different salad? *Hint: we saw that 10 ingredients gives us about 1000 salads.*
- There are about 8 billion (8,000,000,000) people in the world. How many choices does a salad bar have to have to ensure that everybody in the world could make a unique salad?
- When order matters, can you count the possibilities? If you have a lock like the one shown here[3], how many combinations are there when you have 3 dials ordered from top to bottom with each dial having 10 possible values?

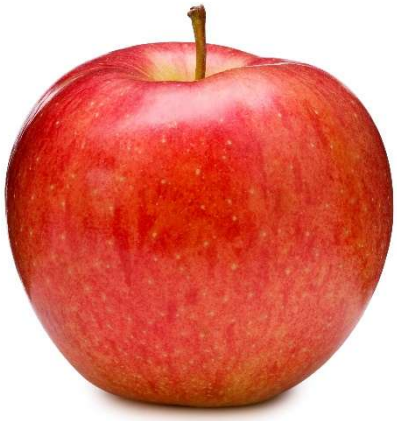
I hope that you had fun! Have a good few weeks!

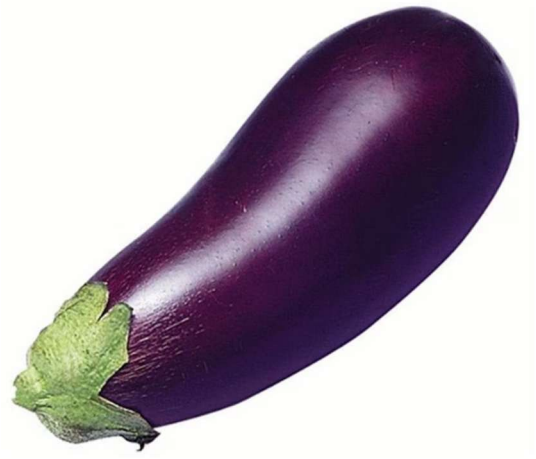
Have Fun!
Mr. Barman

[3] from export.kaiserkraft.com/locks-and-locking-mechanisms/padlocks/combination-lock/165-30-lock-tag-pack-of-6/p/M11776898, accessed Feb. 17, 2024

Appendix: Salad Bar cutouts[4]

Cut these 5 salad bar ingredients out to work on the Salad Bar Problem.





[4] The bean image is from ebay.com/itm/354625857779, the carrot image is from capitalcityfruit.com/shop/carrots-jumbo-5-lb,the dragonfruit image is from shutterstock.com/search/dragonfruits, the eggplant image is from etsy.com/listing/1409772947/florida-market-eggplant-seeds; all were accessed Feb. 16, 2024. The apple image is a freely usable stock image.