The Pool Testing Lab

Warm-up: Cryptograms

In the following "cryptograms" each letter stands for a digit 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9. If a letter appears twice in the same problem, then it stands for the same digit each time. Two different letters in the same problem stand for different digits.

What are the decoded digits?

$$
\begin{array}{r}
M\ E \\
+\quad\ M \\
\hline
A\ S\ A
\end{array}
\qquad\qquad
\begin{array}{r}
S\ U\ P \\
+\ S\ P\ U \\
\hline
U\ P\ S
\end{array}
$$

Part 1: The Lab
Pool Testing (1 infected)
You are all now hired to help in a UNC "Pool Test" Lab. Your first job is to learn what a pool test does.

Suppose someone is infected with a virus, and we need to find out who. If there aren't enough tests to test each person individually, a **pool test** can be used to test many people at the same time.

In this example, three people are tested at the same time.
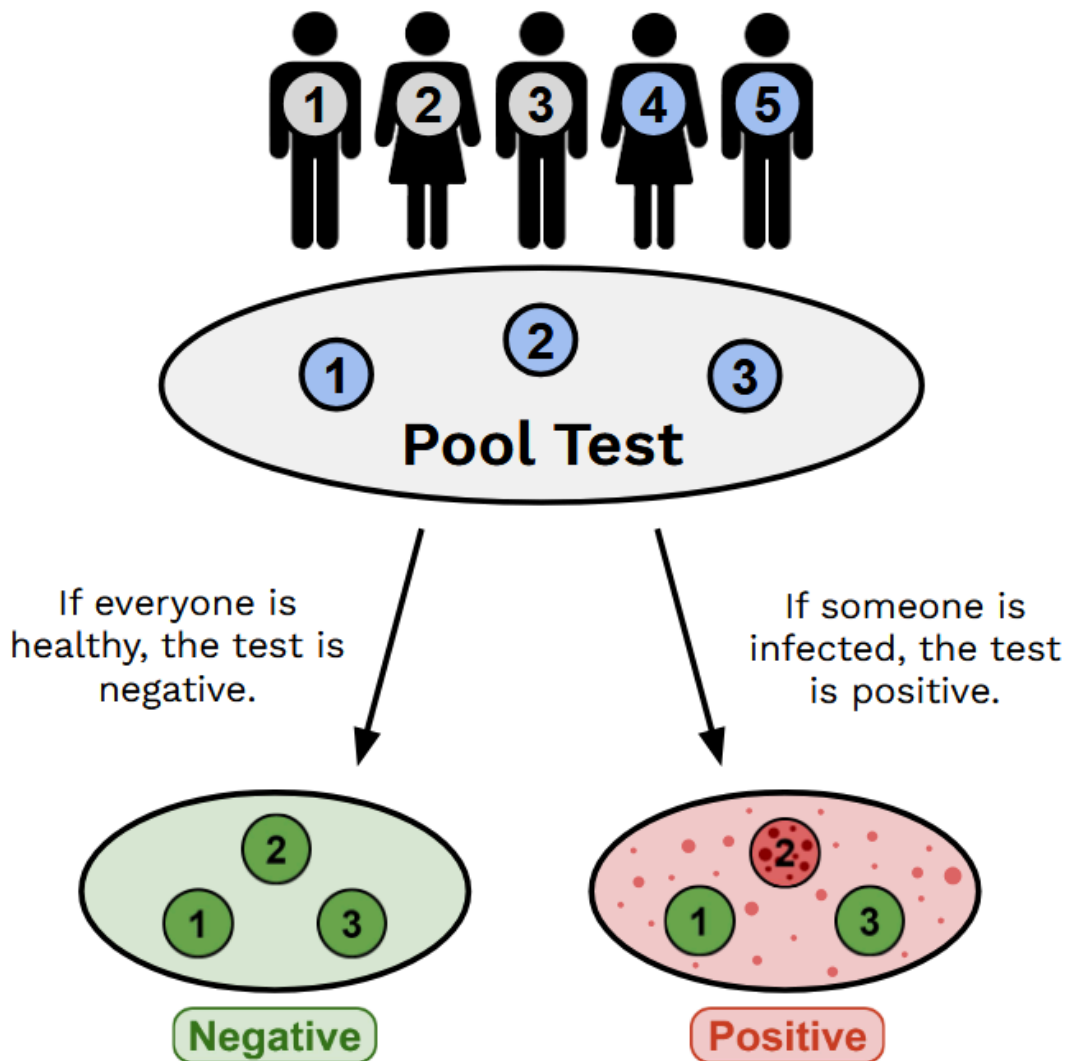
A pool test doesn't usually tell you who is infected, but it does tell you who might be infected.

Lab Task #1:

Three *different* groups of people were tested below. Exactly one person out of each these 5 people is infected. For each group, can you figure out who the infected person is? If not, can you figure out all of the people who might be infected?

Group 1

Group 2

Group 3
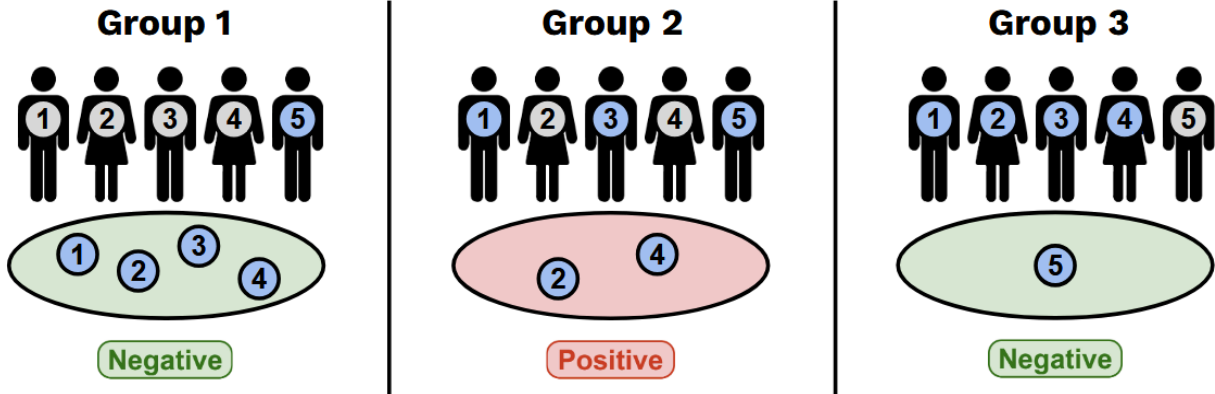
Now, the Lab would like to see your ideas for this next task.

Lab Task #2:

Goal
- Find the infected patient using as few pool tests as possible.

Rules
- Exactly one patient is infected.
- You can do pool tests of any size (including a pool of just one person).
- You will usually need to do more than one pool test to find the infected person. You can test the same patient any number of times.

1) To start, there are 6 patients and exactly 1 patient is infected.
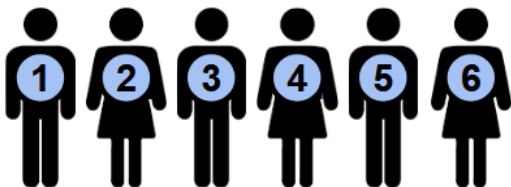
What is the fewest number of tests you need to guarantee that you will always be able to find the infected patient?