Chapel Hill Math Circle
Session 6 – November 8, 2025: Fun with Angles
Beginners' Group (grades 1-3), 10:30-11:30a
Mr. Barman – dilip@trianglemathinstitute.com
Supplies needed: rulers, protractors, scissors, blank and graph paper, colored pencils

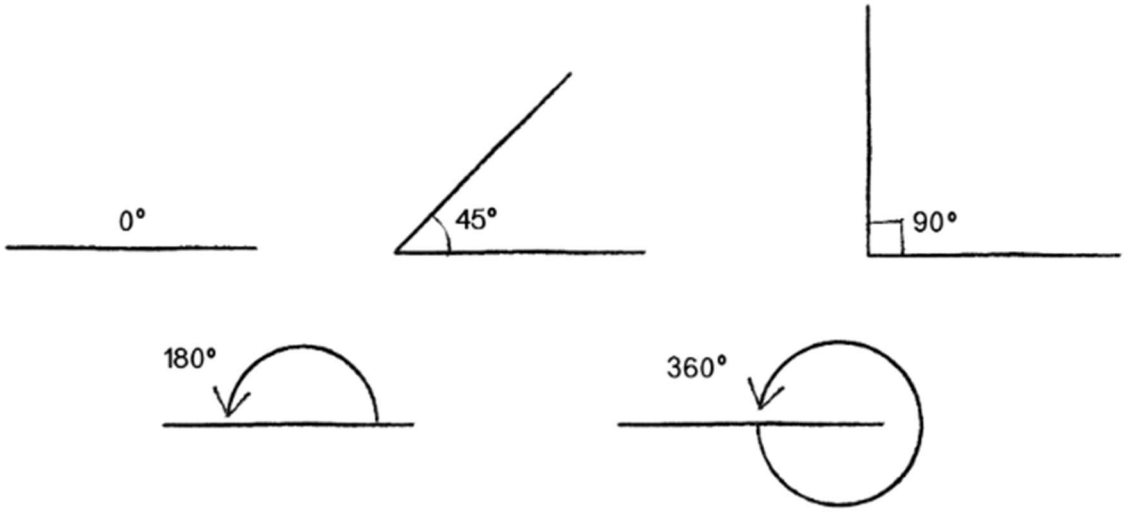
Welcome to Chapel Hill Math Circle! We're glad that you are here to have some fun! Let's pursue angles! By the way, I have added to the last session's handout on diagonals; you can get it at mathinst.com/diagonals.

Angles

What is an angle? Take a minute to discuss with a friend or parent what you think an angle is. Can you demonstrate with your body some angles? Jot down any notes here.

Notes on what an angle is

We will define an angle as the **space formed inside two intersecting lines** or line segments. We typically measure the space by a number called the **degree** that goes from **0 to 360**. Khan Academy[1] has a nice summary image.



[1] khanacademy.org/computing/computer-programming/programming-natural-simulations/programming-angular-movement/a/angles-and-units, accessed Nov. 8, 2023

I like to compare angles to the opening of a book. You might have heard of obtuse angles, acute angles, right isosceles triangles, equilateral triangles, and more. Don't worry about these names for now. And it's okay if you haven't heard of them!

A right angle or 90 degree (90°) angle is like the corners of a piece of paper. Degrees numbered between 0 and 90 measure how open a book is.

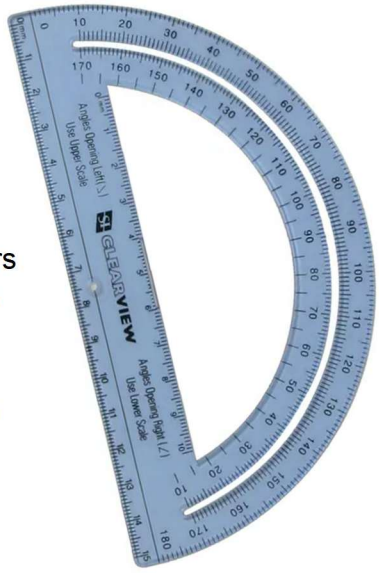
Questions

Why do we use numbers like 90, 180, and 360? What is an angle like 360°+360° = (300+300+60+60 = 600 + 100+20 = 720) 720°? Or 360° added together many times? Please discuss with a friend and jot some notes down. Leave space to add more notes when we discuss this in a few minutes.

Notes on angle questions

Protractors

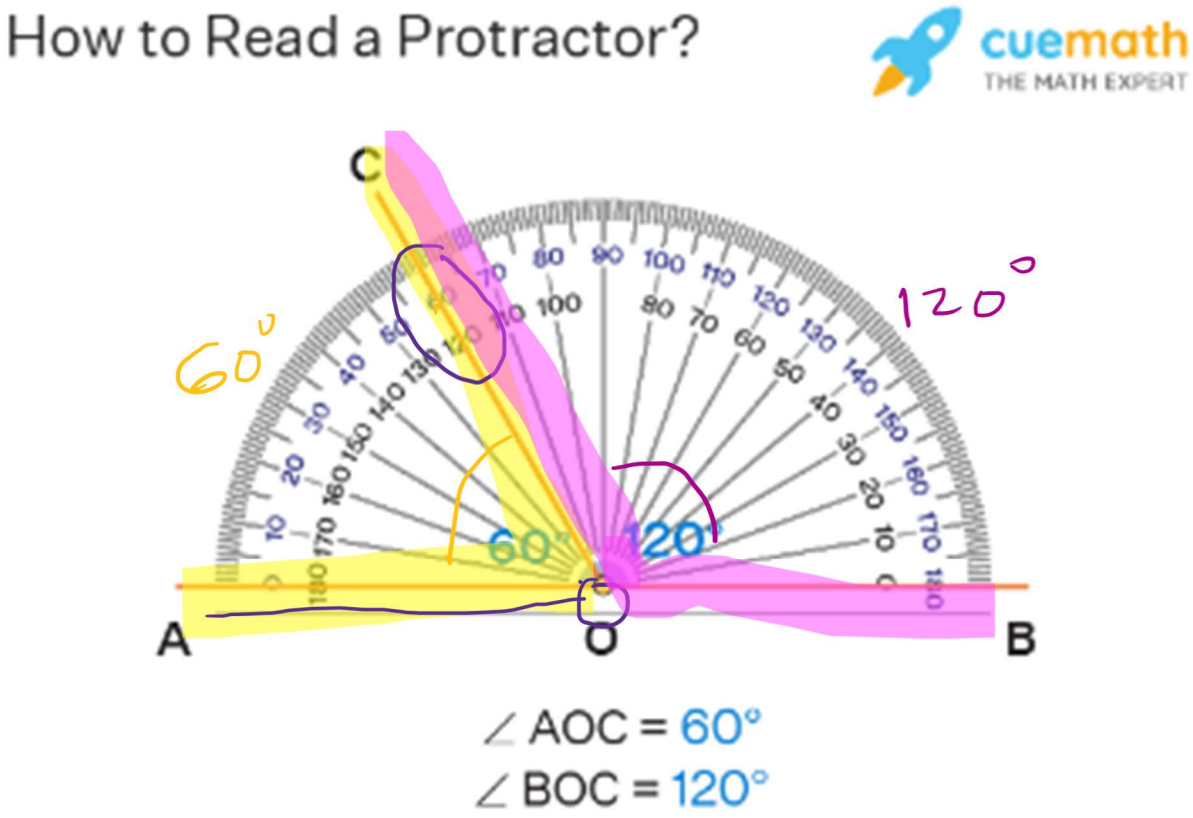
Have you ever heard of a protractor[2]? I will show you how to use it to measure angles with the help of a picture from cuemath[3].



- Use a ruler to make an angle.
- Line up the center hole (marked O below) with the angle's vertex and have one side of the triangle go along the line at the bottom of the protractor (A to O or O to B). Look at what numbers the other line goes through. In the example below, line OC goes through both 60 and 120.
- Angles smaller than a right angle will be between 0 and 90 degrees. Therefore AOC, which is less than a right angle will be 60°. BOC, which is more than a right angle, will be the bigger number, 120°. Note that a straight angle is 180°.

[2] The picture of the protractor is from Fisher Scientific, fishersci.com/shop/products/clearview-transparent-plastic-protractor/S90491A, accessed Nov. 8, 2023

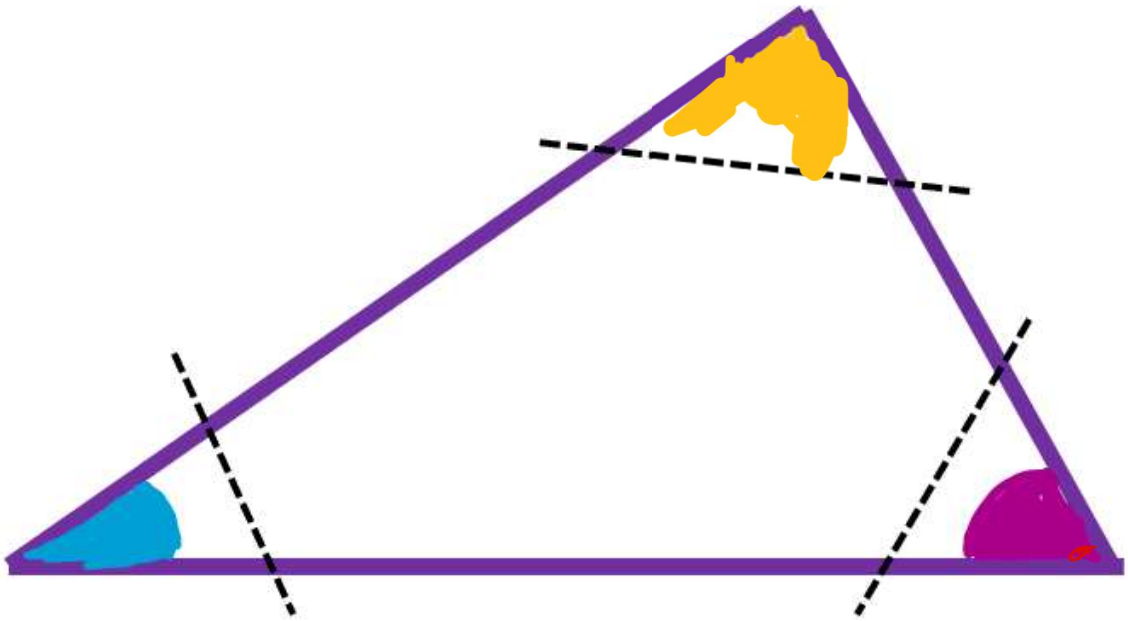
[3] from cuemath.com/geometry/protractor, accessed Nov. 15, 2023



A straight angle is $180°$ … and the angles of a triangle sum to $180°$

As we saw on the previous page, a straight line can be viewed as a $180°$ angle. Make a few triangles. Make them different sizes, with different sizes of angles; try some that have corners like a page (a right angle or $90°$ angle); try some where all the angles are the same; experiment! In each of these, color the three angles different colors like I have below.

Next use scissors to cut out the three angles. You can even just rip; the cuts don't have to be clean. You should have something like the three pieces at the bottom. Shove them together and you will find that no matter what triangle you have started with, you have the three pieces forming a line. This shows that the angles of a triangle add up to a $180°$!



Keep on experimenting

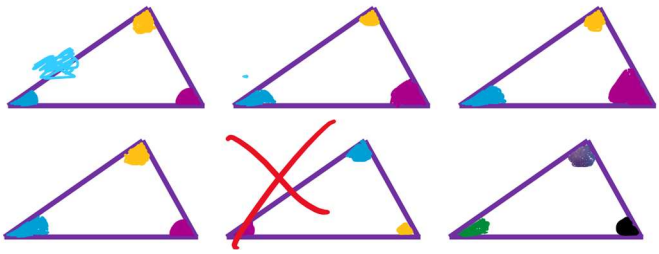
Play with your protractor. Make and measure different angles. What do you think 180° + 90° = 270° looks like? 180° + 180° = 360°?

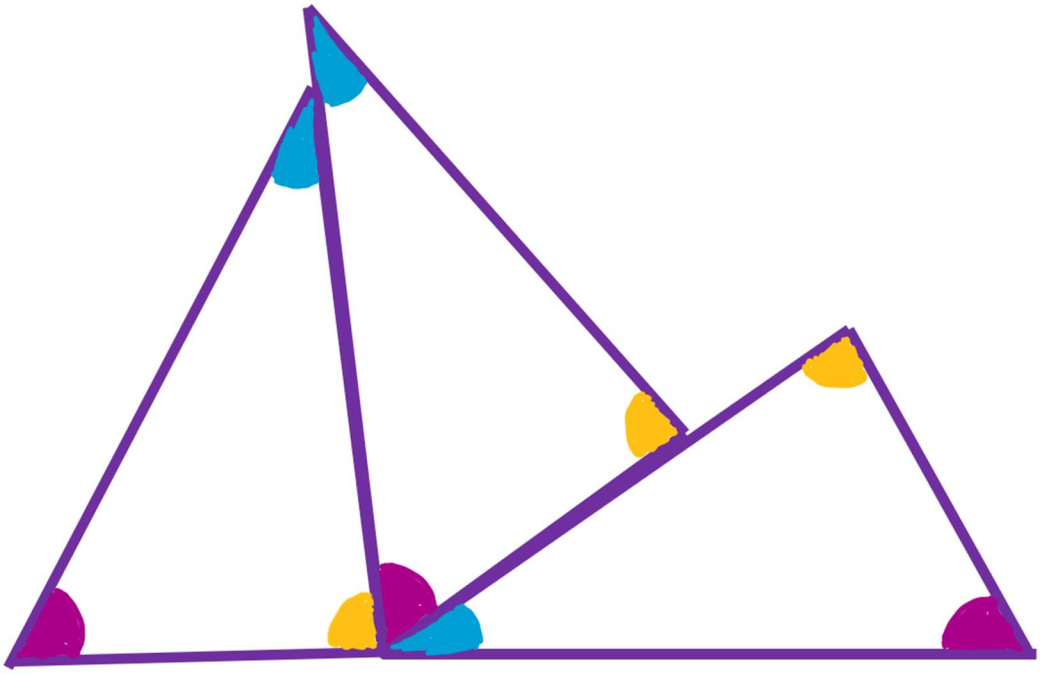
Here is a second way to show that the angles of a triangle add up to $180°$, or a straight line. Make three identical copies of a triangle. Here are a few ways to do this:

- Make one triangle then use tracing paper to make two more copies.
- Find a triangular object and trace around it to make three copies.
- With your family's help, find a printable triangle or have them help you make a printable triangle with a computer. Print 3 copies.

Once you have your three triangles, put them all in front of you so they are arranged the same way. Color each of the three angles in one triangle a different color just like before. Duplicate this for the other two; the amount of coloring doesn't have to be the same but the same angles in each triangle should have the same color.

It should look something like the top and not the bottom example here to the right.
Once you have done this, cut out the three triangles and arrange them so that each of the three colors meet at a point. Do the triangles come together as a line at the bottom? If so, how does this show that the angles of a triangle add up to $180°$?





Play with your protractor. Make and measure different angles. What do you think 180° + 90° = 270° looks like? 180° + 180° = 360°?

Have a good few weeks!

If you have a protractor at home, keep on measuring. If you don't have a protractor, estimate. What fun angles can you find at home and when you're out and about?

Here are a few questions to think about.

The triangle is considered a strong shape. See if you can figure out why. Do you see triangles in roofs, bridges, playgrounds, and other places?

What happens when you add angles? Draw a 45° angle in one color. The pick another color and add on another 45° angle. What is 45+45? It is 40+50 = 90, so label this angle 90°. Continue on to 135°, 180°, and so forth. What do you notice happens after a while?

What do you think 360° looks like? How about a whole bunch of 360° turns like 10 of them (= 3600°)?

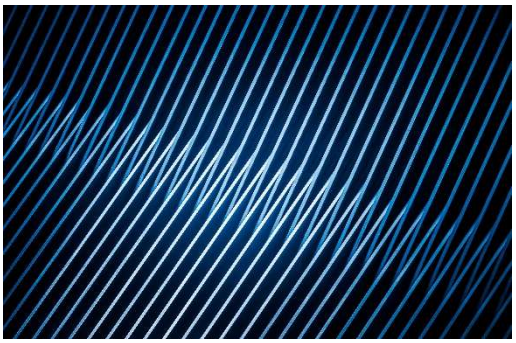
I hope you had fun! See you next time!

Have Fun!
Mr. Barman

P.S. Let your parents know that I host the country's largest vegetarian (all vegan) Thanksgiving! They can visit **trianglevegsociety.org** to learn more. Watch for us on the evening news on Thanksgiving if you don't join us!

Notes for Parents

Angles



Angles and triangles are pervasive. If you have a chance soon or, if not then over the holidays, I'd love for you to take your child to an art museum and explore how artists integrate geometry into their creations. We are lucky to have such great museums such as the Ackland walking distance from math circle, the NC Museum of Art in Raleigh, and Duke's Nasher.

Angles in an n-gon

As a preview, in a future session, perhaps the next one, I hope to have your child use a protractor to measure angles in a polygon. I wanted to have students bridge their technical finesse with their protractors and their intuition to address the problem of angles in an n-gon. The angles of a triangle add up to 180°. If we want to know how many degrees any quadrilateral (4-gon) has, we can readily reason that it consists of 2 triangles and so the angles must add up to 180°x2 = 360°. In general, and I trust your child may find this, an n-gon can include n-2 triangles.

n	# triangles	Total degrees	Avg degree per vertex
3	1	180	180/3 = 60
4	2	180x2 = 360	180x2/4 = 90
5	3	180x3 = 540	180x3/5 = 108
6	4	180x4 = 720	180x4/6 = 120
7	5	720+180 = 900	180x5/7 = 128 4/7
8	6	900+180 = 1080	180x6/8 = 135
N>2	N-2	180x(N-2)	(180x(N-2))/N

To the right is a summary of the sum of the angles and the average angle of several n-gons. We haven't discussed the idea of an average so for now I'll just have the children work on regular polygons.

I don't use multiplication or division with 1st or 2nd graders as I want them to really exercise addition and subtraction. Some of the students may have been exposed to division (yikes!) and can solve the problem of finding the angle of an n-gon by dividing the total degrees by the number of vertices. But I hope that most if not all use a technique of estimating and fixing that is good to know in many contexts.

Thanks for coming to math circle - best wishes!

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