Pigeonhole Principle

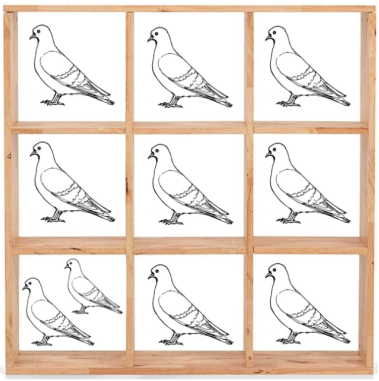
1 Warm-up problems

1. A group of 7 children order ice cream cones from a store that has the following flavors: vanilla, chocolate, strawberry, mint chocolate chip, coconut. Show that at least two of them get the same flavor.

2. There are over ~~64 thousand people in Chapel Hill. Show that at least two of them have the exact same number of hairs on their head.~~ (The average number of hairs on a human head is ~~100,000~~.)

2 Pigeonhole problems

Pigeonhole Principle, Basic Version:
If you put more than n pigeons into n pigeon holes, at least one pigeon hole must contain more than one pigeon.



3. There are more books in a library than there are pages in any one book the library possesses. Must there be two books in the library with the same number of pages?

4. A bag contains several red balls, several blue balls, and several yellow balls. Each day Bex pulls out three balls one at a time, noting their colors in turn. She then returns the three balls to the bag. She does this each day for a month. Prove that, in the month, there were at least two days with exactly the same outcome.

5. (a) How many cards must be selected from a standard deck of 52 cards to guarantee that at least three cards of the same suit are chosen? Suit means hearts or diamonds or clubs or spades.

   (b) How many cards must be selected to guarantee that at least three hearts are selected?

6. Sixteen boxes of chocolate are for sale at the store. The chocolates are of three different kinds (dark, milk chocolate, and white chocolate), and all chocolates in a box are of the same kind. You want to buy 6 boxes of chocolates to give to your 6 cousins, but you want to give them all the same kind of chocolate so there won't be any squabbling. Is this necessarily possible?

7. Twenty-five crates of apples are delivered to a store. The apples are of three different varieties: golden delicious, granny smith, and pink lady. Each crate contains only one variety of apples. Show that there must be at least 9 crates that all contain the same type of apple.