Part 1: Four Points, Two Distances[1]

Consider four points in the plane, located at the corners of a square. How many distinct distances between pairs of points are there, in this arrangement?

Apart from the corners of a square, can you find other ways of arranging four points in the plane, such that there are only two distinct distances between them? How many different arrangements can you find?

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[1]Thanks to Alon Amit for the Four Points Two Distances Problem. See https://affinemess.wordpress.com/2009/01/27/four-points-two-distances/