1 Clocks and Calendars

1. (a) Suppose it's 4PM and someone wants to meet you in 5 hours. When is your meeting time?
   (b) Suppose it's 10AM and someone wants to meet you in 5 hours. When is your meeting?
   (c) Suppose it's 9PM and you have a meeting in 12 hours. When is your meeting?
   (d) Suppose it's 1PM, and you have a meeting in 19 hours. What time is your meeting?
2. (a) 20 minutes after 5:47 is 6:_____
   (b) 37 minutes after 3:52 is 4:_____
3. Today, February 15, 2025 is a Saturday. There are 28 days in February and 31 days in March.
   (a) What day of the week was Feb 1?
   (b) What day of the week will Feb 28 be?
   (c) What day of the week will March 17 be?
   (d) What day of the week will April 1 be?

Doomsday Rule

It turns out that the following dates are always on the same day of the year. We will call that day "Doomsday".

- The last day of February
- 4/4
- 6/6
- 8/8
- 10/10
- 12/12
- 5/9
- 9/5
- 7/11
- 11/7

This year (2025) Doomsday is on a Friday. So all of those dates are Fridays in 2025.

4. Use the fact that Doomsday is a Monday to determine what day the following dates fall on in 2025.
   (a) March 7
   (b) August 10
   (c) September 20
   (d) Halloween
   (e) Christmas
   (f) Thanksgiving
   (g) Your Birthday

2 Modulo

For two numbers A and B, we say that $A \equiv B \pmod{12}$ if A and B have the same remainder when divided by 12.

For example, $15 \equiv 3 \pmod{12}$.

Fill in the blank with a small number: $\_\_\_\_\_ \equiv 21 \pmod{12}$?

For two numbers A and B, we say that $A \equiv B \pmod{5}$ if A and B have the same remainder when divided by 5.

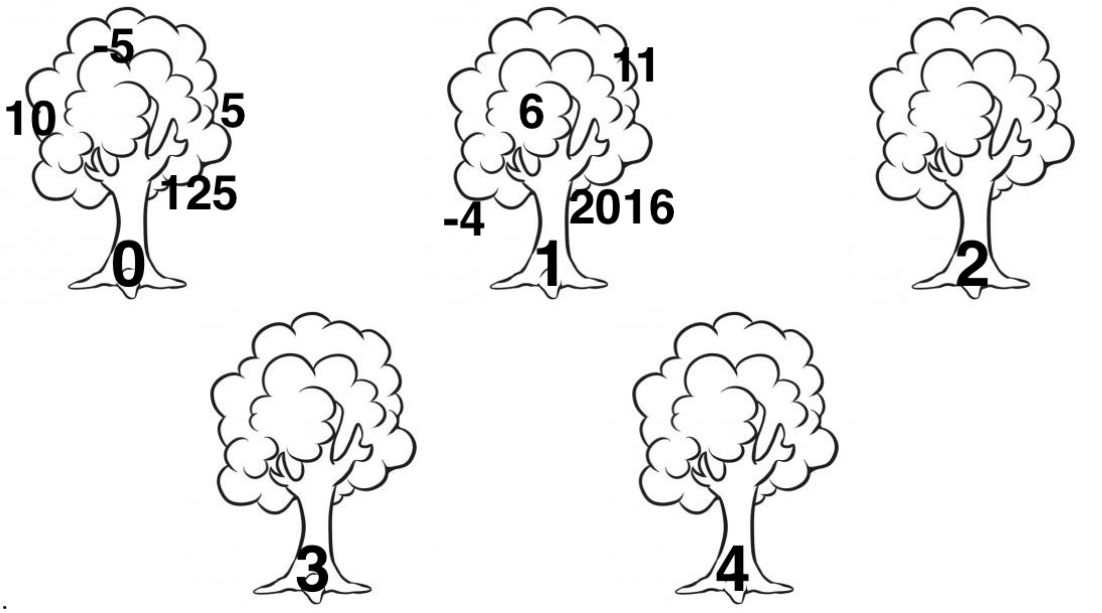
- $8 \equiv 23 \pmod{5}$. Why?
- $8 \not\equiv 14 \pmod{5}$. Why not?

5. (a) Is $13 \equiv 6 \pmod{5}$?
   (b) Is $85 \equiv 0 \pmod{5}$?
   (c) Is $17 \equiv 3 \pmod{7}$?
   (d) Is $5 \equiv 2 \pmod{4}$?
   (e) Is $4 \equiv -1 \pmod{5}$?

3 Mod n Trees

6. Fill in the blanks with the smallest positive numbers possible.
   (a) $52 \equiv$ _____ (mod 12)
   (b) $76 \equiv$ _____ (mod 60)
   (c) $15 \equiv$ _____ (mod 7)
   (d) $15 \equiv$ _____ (mod 3)
   (e) $15 \equiv$ _____ (mod 11)
   (f) $-3 \equiv$ _____(mod 7)
   (g) $-13 \equiv$ _____(mod 5)

7. Here is a drawing of the world (mod 5). On the tree with a 0 on the trunk, we put all the numbers that are congruent to 0 (mod 5). On the tree that with a 1 on the trunk, we put all the numbers that are congruent to 1 (mod 5). Write at least four numbers on each of the other trees.



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8. For the (mod 5) trees above, is it possible to have the same number on two different trees?

9. Draw trees for (mod 2). How many trees do you need? Draw at least four numbers on each tree. What word could you use to describe the numbers on the 0 tree (mod 2)? On the 1 tree (mod 2)?

10. Draw trees for (mod 3).

11. Draw trees for (mod 4). Is there a relationship between the trees (mod 4) and the trees (mod 2)?

4 Adding and Multiplying (mod n)

12. Alicia needs to find $3456+823 \pmod{10}$. Instead of adding $3456+827$, she adds $6+3$. Why does this work?

13. Benjamin wants to calculate $32+67 \pmod{6}$. Instead of adding $32+67$, he adds $2+1$. Why does this work?

14. Carrie wants to calculate $8035+9372 \pmod{2}$. What two smaller numbers could she add instead?

15. Diego needs to find $37 \times 55 \pmod{9}$. What two smaller numbers could he multiply instead?

16. How can you find $35 \times 35 \pmod{36}$ in the laziest way possible?

17. Compute these sums. Hint: you don't need to do a lot of arithmetic.
    (a) $423+577 \pmod{10}$
    (b) $56+89 \pmod{10}$
    (c) $892+9823 \pmod{5}$
    (d) $944+741 \pmod{9}$

18. Compute these products. Hint: be lazy.
    (a) $4893 \times 49024 \pmod{10}$
    (b) $3982734 \times 2398739 \pmod{10}$
    (c) $78 \times 23 \pmod{5}$
    (d) $3874 \times 3284 \pmod{9}$

19. Fill out the addition and multiplication table for arithmetic mod 8.

$+_8$	0	1	2	3	4	5	6	7
0								
1								
2								
3								
4								
5								
6								
7								

$\cdot_8$	0	1	2	3	4	5	6	7
0								
1								
2								
3								
4								
5								
6								
7								

Do you notice any patterns in either of these two tables?