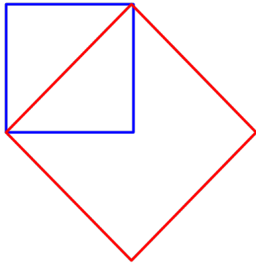


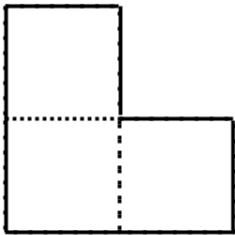
Geometry Puzzles

Chapel Hill Math Circle: Intermediate Group

1. If the area of the small square is 1 square cm, what is the area of the big square?¹

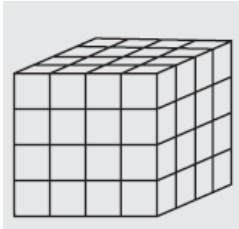


2. The figure shown below can be dissected into three congruent pieces, as shown by the dashed lines. Can you dissect the figure into
- (a) two congruent pieces?
 - (b) four congruent pieces?



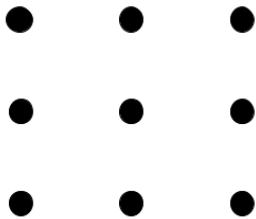
¹Most of these problems are from *Mathematical Circles: the Russian Experience*

3. A cube with sides n cm long is painted on all faces. It is then cut into cubes with sides 1 cm long. If $n = 4$, the cube looks like the picture below. How many of the smaller cubes will have paint on
- (a) 3 surfaces?
 - (b) 2 surfaces?
 - (c) 1 surface?
 - (d) 0 surfaces?



What is the answer when $n = 4$? What is the answer for any value of n ?

4. Draw 4 straight line segments that pass through all 9 points shown below, without picking up your pencil. (The endpoint of the first line segment must be the starting point of the second line segment, etc.)



- 5. Cut a square into 5 rectangles in such a way that no two of them have a complete common side (but they may have some parts of their sides in common).
- 6. Is it true that among any 10 segments, there are always 3 which can form a triangle?
- 7. A king wants to build 6 fortresses and connect each pair of them by a road. Draw a scheme of fortresses and roads such that there are only 3 points where roads cross each other, and at each of these intersections, there are only two roads crossing each other.
- 8. Is it possible to choose 6 points on the plane and connect them by disjoint segments (that is, by segments which do not have common inner points) so that each point is connected with exactly 4 other points?
- 9. Can we tile the plane with congruent pentagons?
- 10. Is it possible to cut a square into several triangles, all of which are obtuse?
- 11. Cut a 3×9 rectangle into 8 squares.
- 12. Prove that a square can be dissected into 2018 squares.
- 13. Cut an arbitrary triangle into 3 parts such that they can be rearranged to form a rectangle.

14. Is it possible to draw a closed 8-segment connected broken line which intersects each segment of itself exactly once?