

Chapel Hill Math Circle

**Chapel Hill Math Circle**  
**Session 7 (last till January) – December 7, 2024:**  
**Analyzing Games and Playing Ultimate Tic-Tac-Toe**  
**Beginners' Group (grades 1-3), 10:30-11:30a**  
**Mr. Barman – [dilip@trianglemathinstitute.com](mailto:dilip@trianglemathinstitute.com)**

*Supplies needed: 1 dice per pair of players*



Welcome to Math Circle session 7! Let's finish the year with some games.

### **Why do people play games? Games of strategy and/or luck**

Children and adults alike have fun with games. When one is sick or needs a distraction, games can help. Competitors play games to get an idea of what each other might do in certain situations. Jot down any other reasons that you can think of why people play games. Many games are those of strategy and/or luck. In October 2024 with 1<sup>st</sup> and 2<sup>nd</sup> graders' at Stanford Math Circle's input, I put together these lists – jot down any games that I may have missed (and let me know).

#### **Games of luck:**

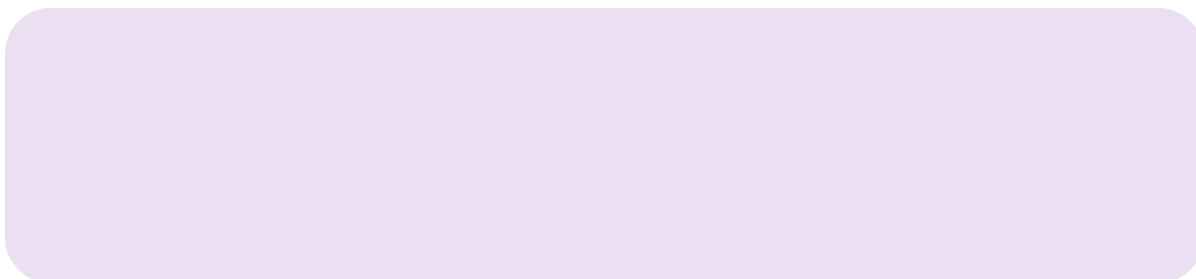
Bingo; coin-flipping games; Candyland; Captain Junior; darts; dice-rolling games; freeze tag; Goblet; Go Fish; Happy Salmon; hide-and-go-seek; HiHo! Cherry-O; Hopscotch; Hungry Hippos; Jumanji; lotteries; Minecraft; musical chairs; number guessing with no hints; Old Maid; Phase; Rock-Paper-Scissors; roulette; SET; Slap Jack; Snakes and Ladders (or Chutes and Ladders); Sorry; Spy Alley; Subway Surfers; Taco Cat Goat Cheese Pizza; tag; "that game at fairs where you just randomly take a duck and see what you will get" [Maxwell, 2<sup>nd</sup> grader in section C]; Too Many Monkeys; Zingo; Zombie Kittens

#### **Games of strategy:**

Black Jack; Blockus; Cellulose; Checkers; Chess; Chinese Checkers; Connect Four; Five-in-a-Row; Gaga Ball; Go; Guess in Ten; Guess Who; Icons; Jumping Chess; Mancala; Memory; Mummy, Mummy, Come Alive; Mysterium; Nim; Prodigy; Qi; Rush Hour; Sequence; Spot It!; Sudoku; Taboo; Tennis; Tic-Tac-Toe; video games; Walls and Warriors; Wei; Wordle

#### **Games of both luck and strategy:**

7 Wonders; Abduktion; Aeroplane Chess; Azul; Backgammon; Baseball; bowling; Candyland; Carcassonne; Catan; Coral Reef; Cover Your Assets; Crazy 8s; Dix It; Dominion; Doomlings; Dragon's Breath; Dragonwood; Dutch Blitz; Exploding Kittens; Fei Xing Qi; Football; Genius Square; golf; Go Nuts for Donuts; Hangman; Happy Little Dinosaurs; Hill Climb; Hoot, Owl, Hoot; Jenga; Liar's Dice; Ludo; Mastermind; Monopoly; Old Maid; Outfoxed; Pokemon Labyrinth; Poker; Quacks and Co.; Qwirkle; Red Light-Green Light; Risk; Rummikub; Scrabble; Shut the Box; Sleeping Queens; Soccer; Splendor; Sushi Go!; Tennis; Ticket to Ride; Trouble; Turn it Over; Uno; Ultimate Tic-Tac-Toe; Ultimate Werewolf; water polo; Wingspan



**Do you always play to win? Worthy opponent assumption**

I bet sometimes, maybe if playing with a young child, you might let your opponent win. When we talk about analyzing games, we assume that all are smart and trying to win. I call that the worthy opponent assumption.

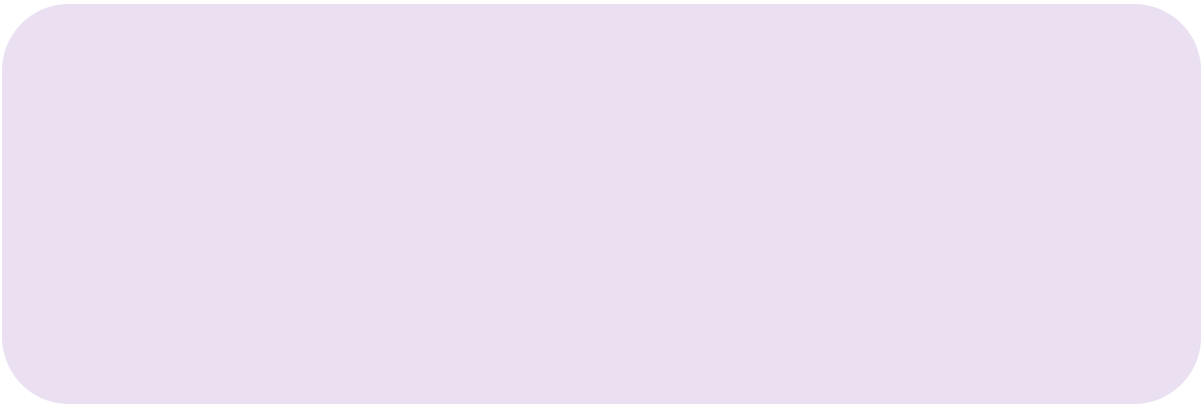
**Analyze this dice game**

Try this game. You take turns with another player. Each person starts with a score of 10. On your turn you can roll a dice or skip. If you roll the dice and get a number that is larger than your score, you do nothing but your opponent can subtract 1. Otherwise you subtract the roll from your score. The first person to get a score of 0 wins.

For example the game might proceed as follows.

	Start	A rolls 6	B rolls 3	A rolls 5	B rolls 4	A rolls 3	B skips	A rolls 4	B skips	A rolls 2
A's score	10	$10-6 = 4$		No chang e		$4-3 = 1$				
B's score	10		$10-3 = 7$	$7-1 = 6$	$6-4 = 2$			$2-1 = 1$		B wins

What might be a good winning strategy? Try playing with a partner for a few minutes and then see if you can come up with a strategy.



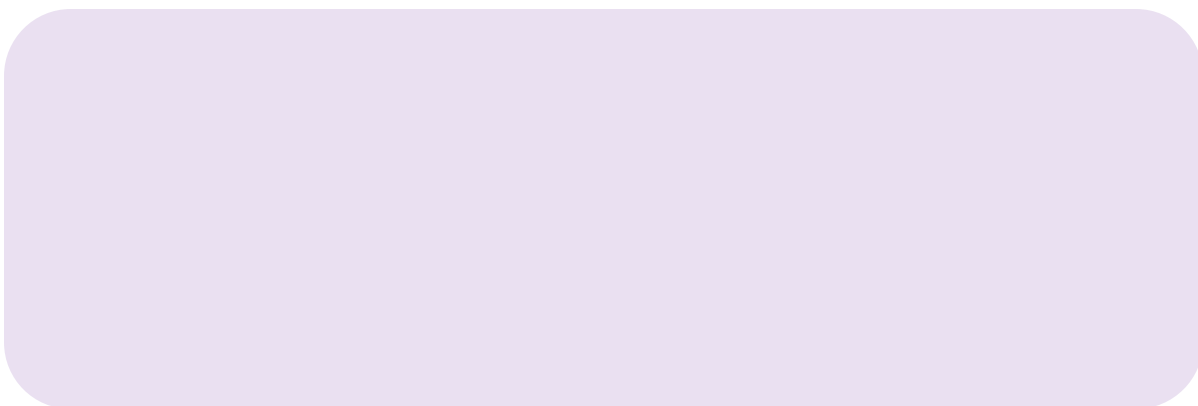
Maybe in the spring we will talk more about games with dice and look into how likely some rolls are.

## Tic-Tac-Toe

All of you have probably played tic-tac-toe. Take a few minutes to play with an opponent and keep track of how many games each of you wins. Do you have a winning strategy?

## Winning strategies

Some games really don't have winning strategies. They can be based purely on luck or on your opponent making a mistake. Can you think of examples of games like these where there are no winning strategies?



## Ultimate Tic-Tac-Toe

There are more versions of Tic-Tac-Toe. Let's look at one called Ultimate Tic-Tac-Toe<sup>1</sup>. Take a look at the grid on the next page for playing.

This game is a bit more complex; you put your mark of X or O where you want initially. Let's say you put it in the very center board and there in the top right corner. The top right corner forces your opponent to play in the grid that's in the top right corner. If they put their mark in the top right corner of that top right board, your turn stays in the top right board. When a player makes a tic-tac-toe in one of the 9 grids, they "claim" that square.

Your goal is to make a set of 3 claims as usual. If at any time you end up in a grid that's already been claimed, you still play there but the winner of that grid won't change. If you end up on a full grid, you get to pick any grid to mark in. Try it! I've not been able to analyze it – can you? Happy 2025!

Enjoy  
Mr. Barman

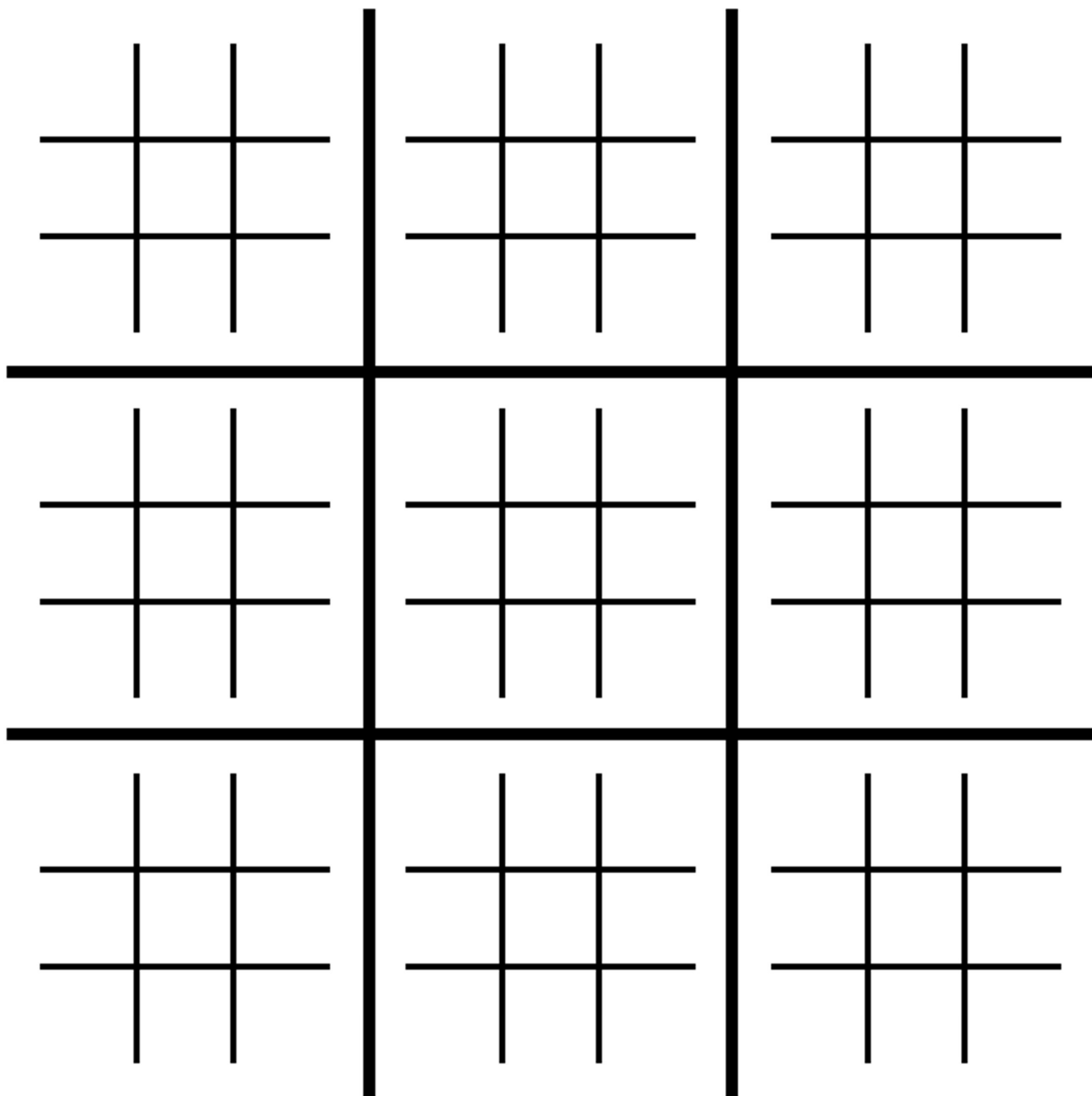
---

<sup>1</sup> This grid is from [mathequalslove.net/ultimate-tic-tac-toe](https://mathequalslove.net/ultimate-tic-tac-toe), accessed Oct. 22, 2024.

# ULTIMATE TIC TAC TOE

**GOAL:** Win three games of tic tac toe in a row.

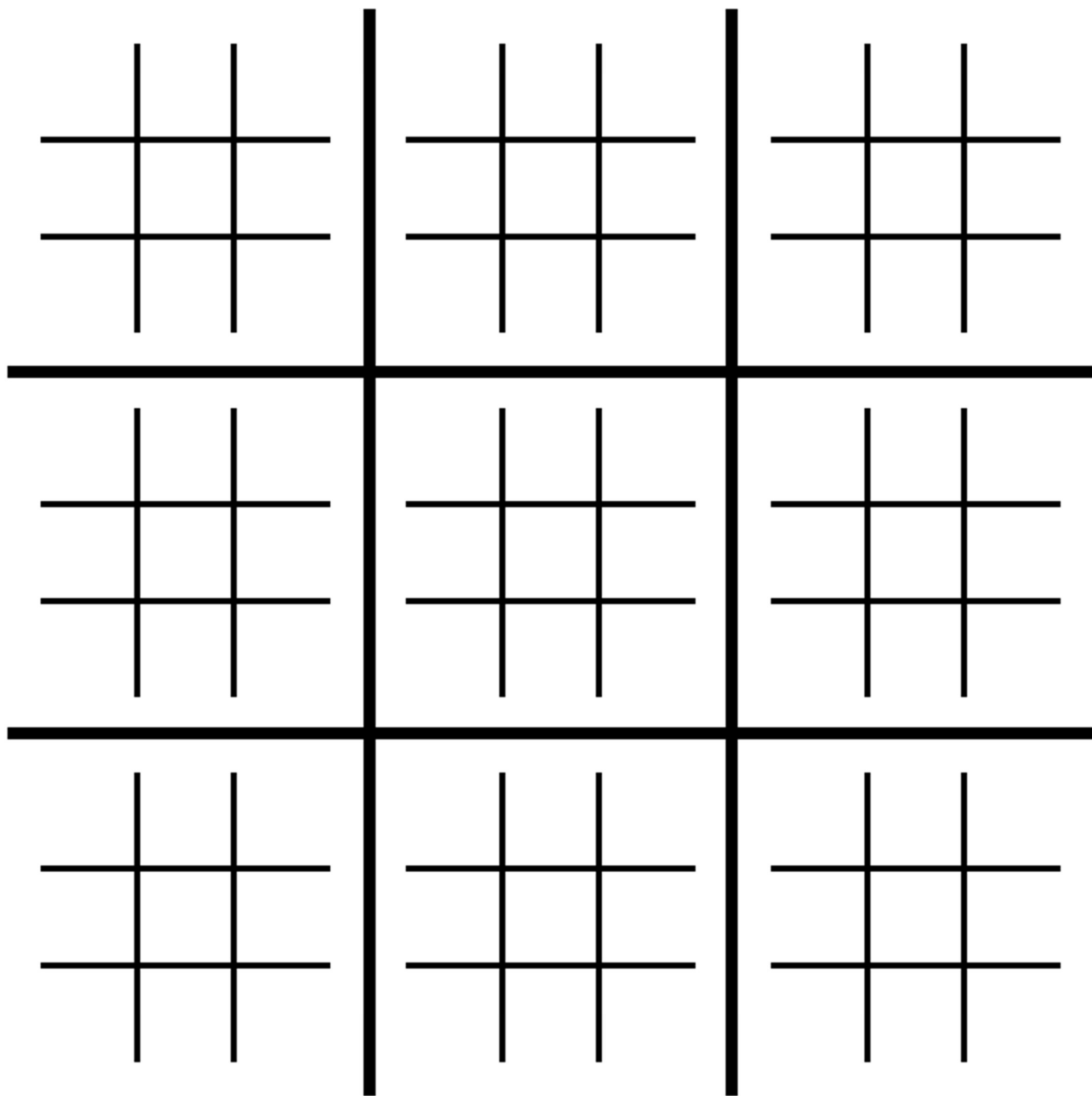
**INSTRUCTIONS:** You may only play in the tic tac toe board that corresponds to your opponent's last play. If you are sent to a tic tac toe board that is already decided, you can choose to play anywhere on the board.



# ULTIMATE TIC TAC TOE

**GOAL:** Win three games of tic tac toe in a row.

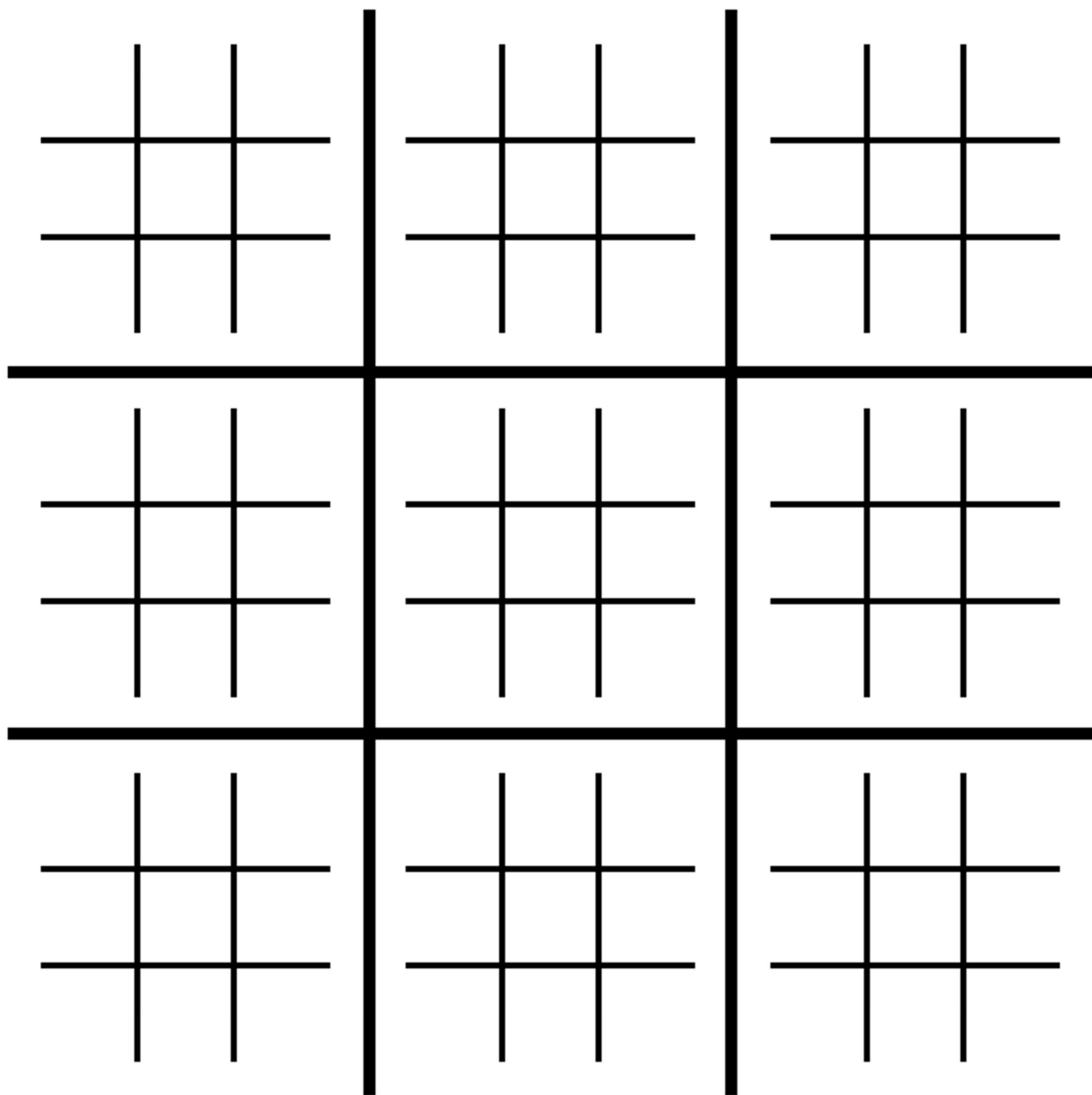
**INSTRUCTIONS:** You may only play in the tic tac toe board that corresponds to your opponent's last play. If you are sent to a tic tac toe board that is already decided, you can choose to play anywhere on the board.



# ULTIMATE TIC TAC TOE

**GOAL:** Win three games of tic tac toe in a row.

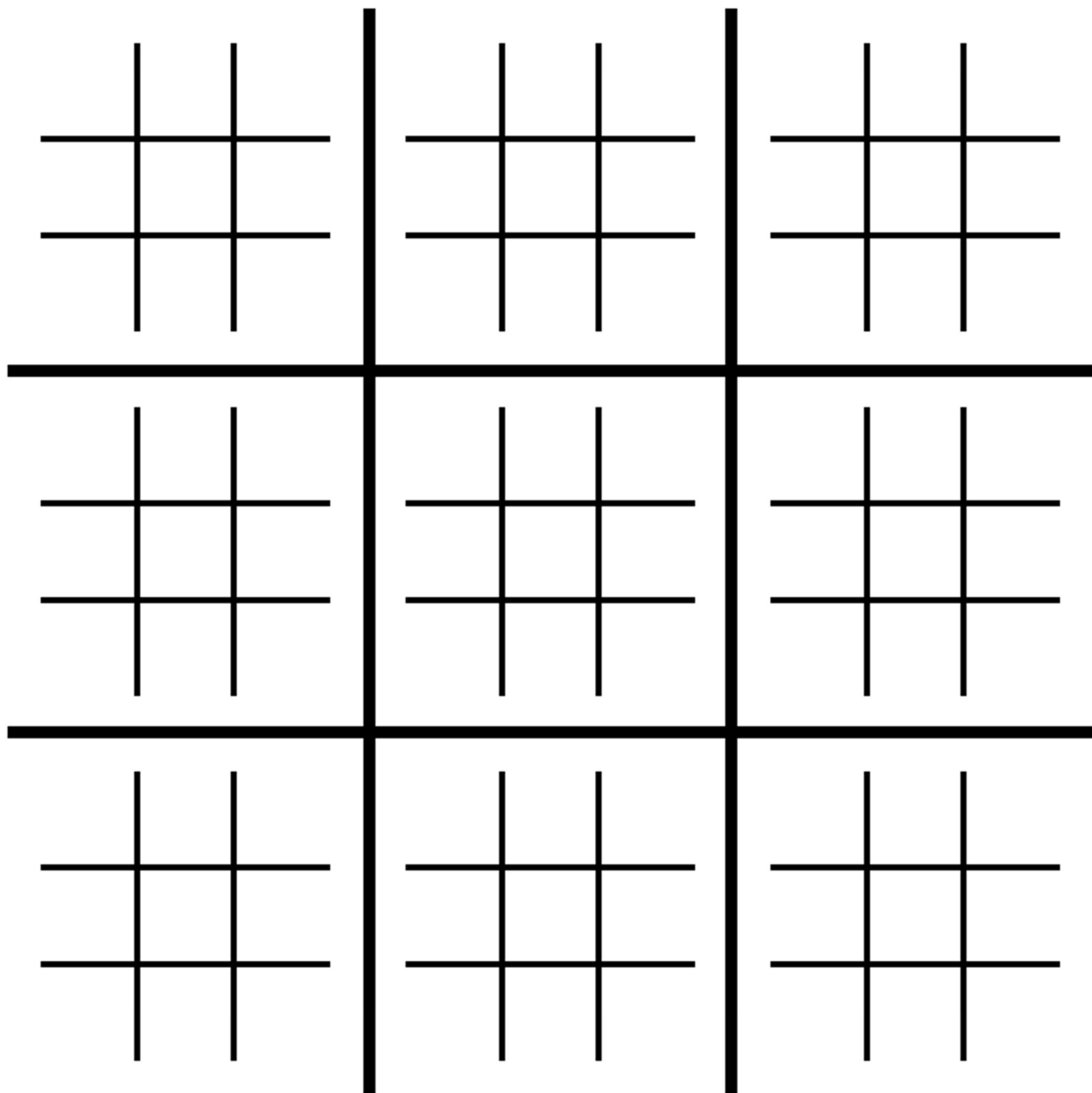
**INSTRUCTIONS:** You may only play in the tic tac toe board that corresponds to your opponent's last play. If you are sent to a tic tac toe board that is already decided, you can choose to play anywhere on the board.



# ULTIMATE TIC TAC TOE

**GOAL:** Win three games of tic tac toe in a row.

**INSTRUCTIONS:** You may only play in the tic tac toe board that corresponds to your opponent's last play. If you are sent to a tic tac toe board that is already decided, you can choose to play anywhere on the board.





## Notes for Parents

Games are a rich area for enjoyment. Beyond the obvious pleasure and distraction that they can provide, they encourage creative problem solving which is the basis of mathematics. Game theory is a branch of mathematics that is valued by militaries and corporations where opposing forces hope to prevail.

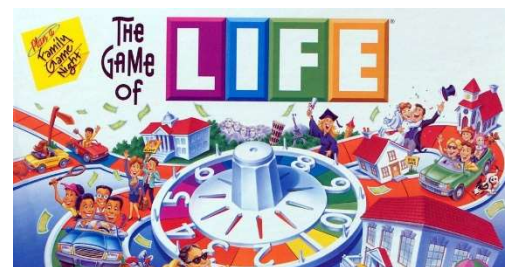
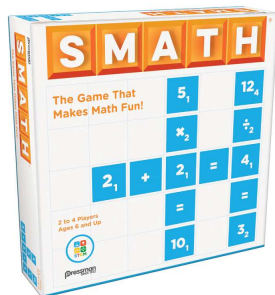
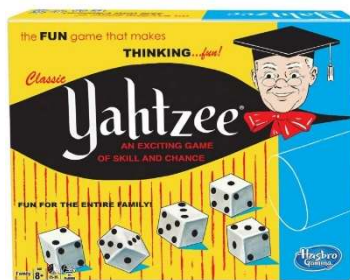
Tic-tac-toe is rarely played beyond early elementary school because it really isn't winnable. I was introduced to ultimate tic-tac-toe at a math teachers' summer camp. A group of perhaps 30 of us math teachers played but didn't come up with some strategies to guarantee winning. There are other versions of tic-tac-toe such as a 3-dimensional one to encourage topological thought, and a circular one.

Thank you for attending the Chapel Hill Math Circle sessions. It's my pleasure to work with the children. We will start up again in January. In the meantime, I hope that your family will enjoy the holidays and time off with appreciating and enjoying how math is all around us – in the shapes of fruits, the ways of making change at the holiday market, the ways of taking all's preferences into account in making decisions, ... and in play<sup>2</sup>!

Till next time!

*Dilip*

Dilip Barman dilip@trianglemathinstitute



<sup>2</sup> All images copied on Oct. 23, 2024. Spot It!: [amazon.com/Zygomatic-Spot-Original-Party-Game/dp/B07NQLG6B3](https://amazon.com/Zygomatic-Spot-Original-Party-Game/dp/B07NQLG6B3), Life: [boardgamegeek.com/boardgame/2921/game-of-life](https://boardgamegeek.com/boardgame/2921/game-of-life), Yahtzee: [learningexpress.com/z41017/classic-yahtzee.html](https://learningexpress.com/z41017/classic-yahtzee.html), Battleship: [shop.hasbro.com/en-us/product/battleship-classic-board-game-strategy-game-for-kids-ages-7-and-up-fun-kids-game-for-2-players/54D1C85E-CFBE-4625-9A9E-53C36F4D136C](https://shop.hasbro.com/en-us/product/battleship-classic-board-game-strategy-game-for-kids-ages-7-and-up-fun-kids-game-for-2-players/54D1C85E-CFBE-4625-9A9E-53C36F4D136C), SMATH: [areyougame.com/products/pr5202-06-smath?variant=40472797708457](https://areyougame.com/products/pr5202-06-smath?variant=40472797708457), SET: [playmonster.com/product/set](https://playmonster.com/product/set)