Chapel Hill Math Circle
Session 3 – October 12, 2024:
Estimating the World Around Us part 2
Beginners' Group (grades 1-3), 10:30-11:30a
Mr. Barman – dilip@trianglemathinstitute.com

Supplies needed per person: access to outdoors

Welcome to Math Circle session 3! If you were with us last time we did a unit on estimating – we focused on population and whether population numbers are really knowable. We also looked at what things are easy to count, what are difficult to count, and what are impossible to count.

This time we may go outside! We will learn some techniques called **Fermi estimation**, named after physicist Enrico Fermi[1] (1901-1954). Let's look at some math that can be helpful and then go outside and count somethings! We can stay inside if people want and count by looking out the windows or count things in the building.

Order of magnitude estimating

Last time we considered, "according to the 2020 Census (data.census.gov), there are 61,960 people living in Chapel Hill, 21,295 in Carrboro, 283,506 in Durham, 174,721 in Cary, 58,780 in Apex, and 467,665 in Raleigh." We agreed this is too much information, even if the numbers are correct. Isn't it enough to know that there are about 60,000 in Chapel Hill, 20,000 in Carrboro, 300,000 in Durham, 200,000 in Cary, 60,000 in Apex, and 500,000 in Raleigh? Even better, there are a bit over a million people in our area?

If you take a trip to an area, how would your expectations (about things like availability of museums, airports, restaurants, public transportation, playgrounds, and similar) be different if you're going to a state like Montana that has about a million people (in the whole state!) or a city like New York that has about 8 million?

One way of counting is by "order of magnitude". How many decimal digits does a number have?

- Numbers like 0-4 are not far from 0
- Numbers like 5-14 aren't far from 10
- Numbers like 15-24 aren't far from 20
- …

But we can be more rough. We might measure:

- The number of slices of pizza that you can eat in a day by a single digit
- The number of strawberries that you can eat by two digits
- The number of math problems that you can do in a week by three digits
- The number of items of mail that your family gets in a year by four digits

[1] Picture of Enrico Fermi courtesy Argonne National Laboratory; accessed Oct. 11, 2024 from flickr.com/photos/argonne/5038837435/in/photostream.

Let's look at that last example. If you were to guess, how much mail does your family get each day? I bet it's in the low 2-digits – maybe 10? Probably not 2 or 3, and probably not 20. Let's call it 10. There are about 365 days in a year; even if we ignore that some days don't get mail delivery, if we get 10 pieces of mail 365 times a year, would we get 3650 pieces of mail? Maybe your family gets a lot of mail – 20 pieces a day? That's still less than twice 4000 or 8000, which fits in 4 digits.

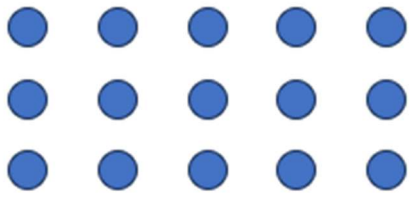
This kind of counting is called **order of magnitude** counting or estimating. Give it a try. Using only numbers starting in 1s and ending in 0s (10, 100, 1000, 10,000, 100,000, 1,000,000, and so forth), can you estimate how many apples you eat in a month, how many days of rain there are, how many socks your family has, how many books your local library has, and anything else? These are guesses but you probably won't be way off. If you really want, you can be just a little bit more specific, like about 20 or 30, or about 500 or 600.

Tools to estimate

If you notice that a plant has leaves in groups of 12 and then you do an order of magnitude estimation that there are about 100 such groups, what is your estimate of the total number of leaves? We do a lot of calculations like this. There are three general rules that we use:

1. Remember that we are estimating so **don't take too long to come up with an estimate**; it's okay to be "wrong" but if you think about something you can observe or know about, you probably aren't going to be very off.
2. **Use easy calculations** – after all, our numbers are (hopefully good) guesses. Instead of 98 + 102 + 110 + 80, realize these are all around 100 so make it 4 100s or 400. Instead of 37 or so petals per flower and something like 61 flowers, call it 40 petals per flower and 50 flowers; this is like 20 petals per flower with 100 flowers or about 2000 petals.
3. **If there are 10 of something we just add a 0**; 10 2s is 20. **If there are 100 of something we add two 0s**; 100 5s is 500. That pattern holds for larger orders of magnitude.

Let's make sure that we understand the second point. If we find leaves in clusters of 8, why do 100 clusters have 800 leaves? All we are doing is skip-counting. We need to skip count by 8 100 times. But this is the same as skipping by 100 8 times. Does this picture show why that's the case? Here to count to 15, we can skip by 3 5 times or by 5 3 times.



So 100 8s (8, 16, 24, 32, …) is the same as 8 100s, which is much easier to count (100, 200, 300, …, 800). Do these calculations make sense?

- 13 10s is 130
- 10 13s is 130
- 142 10s is 1420
- 5287 100s is 528,700
- 367 1000s is 367,000

Let's use the rest of our time to do estimating. Let's go outside and estimate leaves in trees (use your creativity – for example, try to carefully estimate the leaves on one branch and then estimate the order of magnitude of branches), maybe grass in a lawn (estimate how many blades are in 1 square inch and then estimate how many square inches there are), or something else. We can also work inside; how many windows are there in the building? How many pieces of chalk are there? How many people are attending Math Circle?

In a future class we might look at other problems like estimating things like:

- How many slices of pizza will be eaten in the U.S.A. today? How would it help to know that there are about 333 million people in the country?
- How many dosas (a South Indian specialty) will be eaten in the Raleigh-Chapel Hill-Durham "Triangle" area of North Carolina today[2]?
- How many people will read *Winnie the Pooh* today across the world?
- How many onions are harvested a year in France?
- How many vegans are there in Kentucky?

Some of these questions will need you to know some information (what is the population of a country, how many vegans are there, etc.). Part of being a smart person is knowing which questions to ask. Have fun – keep on estimating!

[2] What questions would you want to ask? Would it help to know about how many people of South Asian background live in the area? Could you estimate it from the population of the country if you knew that about 1 or 2 out of every 100 people are of South Asian ancestry?

Notes for Parents

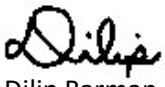
If your child was at our earlier estimation session then I hope that it inspired a sense of numeracy. This session is independent of that one; the idea of thinking about things that nobody except possibly a divine being or omniscient universe can count and having some tools to reasonably estimate I hope will feel empowering.

I first learned about Fermi estimations in college. As a Freshman, I took a physics seminar that met once a week for fun to discuss things like this. I am so glad that I took this class!

Fermi estimating is also called "back of the envelope calculating". I suspect you do it reasonably often. Perhaps you get four estimates for painting your house that vary from $4932 to $7800. You discard an outlier low bid and estimate that with some overage and unexpected costs that the job is about $8000. You have saved $115/week dedicated to home improvements over the past year and are getting a CD coming due that is worth $4000 that has paid 4.75% interest over a year, all payable at the end. Your "back of envelope" calculation might be that your CD is worth something like $200 (10% of $4000 is $400 so 4.75%, close to 5%, is about $200) interest plus $4000 capital = $4200, so you have to come up with about ($8000-$4200 is about $4000). You have saved something like 52 x $115 which is about 50 x $120 which is about 100 x $60, or $6000. So you have about $2000 to spare!

A few good references for Fermi approximations are LessWrong's "Fermi Estimates"[3] and Brilliant's "Fermi Estimate"[4]. Innovative Teaching Ideas has a nice set of problems for teaching, "An Excellent Collection of Fermi Problems for your Class"[5].

Till next time!



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P.S. I love teaching fun math classes at a school that I helped to open, trianglemathinstitute.com. Several of our classes this year are full or nearly full, but we have space especially in 3rd grade and geometry. We are thinking of opening a second prealgebra section and possibly offering a precalculus or a calculus class. Let me know if you are interested.

Mathematics Institute of the Triangle

[3] lukeprog, 11 Apr 2013, lesswrong.com/posts/PsEppdvgRisz5xAHG/fermi-estimates
[4] July Thomas et.al., brilliant.org/wiki/fermi-estimate
[5] Kevin Cummins, ed., 14 Aug 2023, innovativeteachingideas.com/blog/an-excellent-collection-of-fermi-problems-for-your-class