Shapes of Revolution for Winter Holidays

PATHWAYS
#26 Real Functions
#28 Measure and Integration
#51 Geometry

Family Math Circle ~~~ December 2023

Pathways name our math inspirations using Mathematics Subject Classification. Mathematicians around the world label their work with library codes from MSC, https://msc2020.org/ Let us help students feel happy familiarity with each subject area!

Family Math Circle is an informal learning space where participants make advanced mathematics accessible to everyone in kind ways.

Roots and Wings
Make models to ground your math. Make connections to advance in higher math.

Turn around, swing objects around, and rotate ropes to feel the geometry of circular motion in space. Capture the motion: Make paper ornaments with shapes of revolution to model functions and integration.

Words With Math Friends
Tell friends and family all about your math creations. Use these terms +"math" to find images, videos, and articles on the web.

- point, line, curve, surface, solid, dimension, 0D-1D-2D-3D
- to rotate, to revolve, surface of revolution, body of revolution

- circle, disk, ring, sector, shell
- line symmetry, radial symmetry
- infinity, infinitely large, infinitesimal
- length, area, volume, measure, integral

Interesting Choices
Mathematicians do many different things. What kind of math person will you be today?

Go on a scavenger hunt for shapes of revolution or rotate something to see what shapes emerge. Start with geometric shapes and explore their measures and functions, or rotate a function you know. Create your own unique shapes of revolution or model jingle bells, lanterns, balls, icicles, spinning tops, etc.

Toolbox

Physical (gray=optional)
Rope or string, paper, colored pencils, scissors, glue, stapler, pebbles or chips, colored paper, graph paper, pipe cleaners.

Virtual
GeoGebra 3D with AR https://www.geogebra.org/m/qbxbcmqw
GeoGebra jingle bell https://youtu.be/3HDTDwsPIb0

NaturalMath.com/circles/

Extra activities, videos, math connections, books, and other resources for math circle leaders.
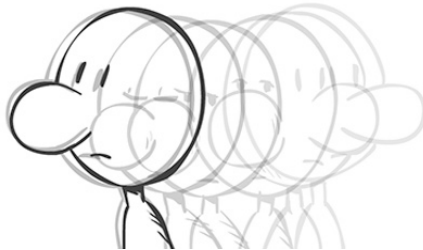
Ornaments: Thornmeadow Glassworks

Shapes of Revolution for Winter Holidays

0. Warm-up: Turn It Around!

Find something that you can move around and around. Or someone! What parts of your body can you rotate? Try and find out. Can you find 5 more examples of things that you can rotate? 10 more?

Images: Daxiao Productions, Pilar Newton, Dolittle and Howell

1. Your Math Superpower: Time-Lapse Vision

Think of these experiments as training a mathematical superpower: time-lapse vision! Ask a math friend to hold one end of a jump rope or string in place. Hold the other end and step aside until the string is pulled tight. Take a box with small objects such as game chips or pebbles. Walk around your friend, keeping the string tight. Leave a pebble on the ground every step. Look at the pebbles together. Activate your time-lapse vision and imagine the pebbles along a curve. What curve is it?

> ⚠️ **SPOILER** Pebbles or dots model points. We call points zero-dimensional objects (0D) because we can't move in any direction within a point. We call lines or curves one-dimensional objects (1D) because we can move forward or backward along them, as if along one single road.

You can also rotate a pen or a piece of chalk on a smaller string and draw dots every so often, or leave footsteps in the sand or in the snow. Imagine smaller and smaller steps, with more and more pebbles along the curve—or infinitely small (infinitesimal) steps and infinitely many pebbles adding up to form the curve.

Time-lapse photo of the sun by Pekka Parviainen and the swing by Chris Frank for Getty; Walking Around by John Cumming

2. The Line and the Lantern

Take a pipe cleaner or a narrow strip of paper and bend it. Hold one end and ask a math friend to hold the other end. Hold the ends very still, and rotate your strip. Your curve will trace an invisible surface in space. Use your time-lapse vision. Can you see that surface in your mind? Here are some surfaces of revolution modeled with computer tools:
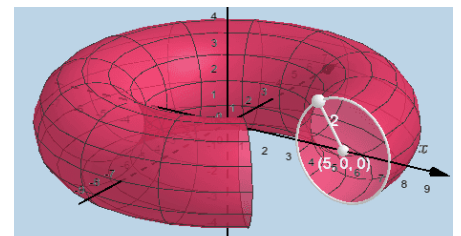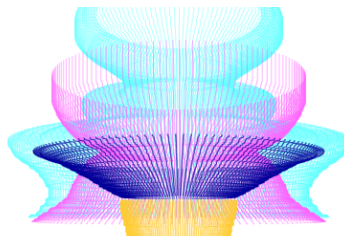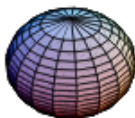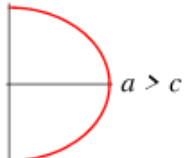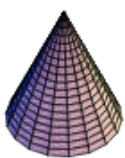
Good news: You can hold these beautiful mathematical objects in your hand! Or rather, beautiful paper models of the mathematical objects. Add up (integrate) multiple strips of paper around a circle, then glue them in place. Try folding paper into different curves (before or after cutting) to get different surfaces.

Lanterns by Origami Resource Center; a torus by Polyhedr.com; spheres by Mod Podge Rocks

3. The Triangle and the Tree

What if you rotated a flat (2D) shape instead of a curve? Let's try! Draw a shape with a straight side. Tape the straight side to a pencil, like a flag on a stick, and rotate the pencil around itself. Your shape would travel inside an invisible figure in space. Here are some virtual models of such solids of revolution.
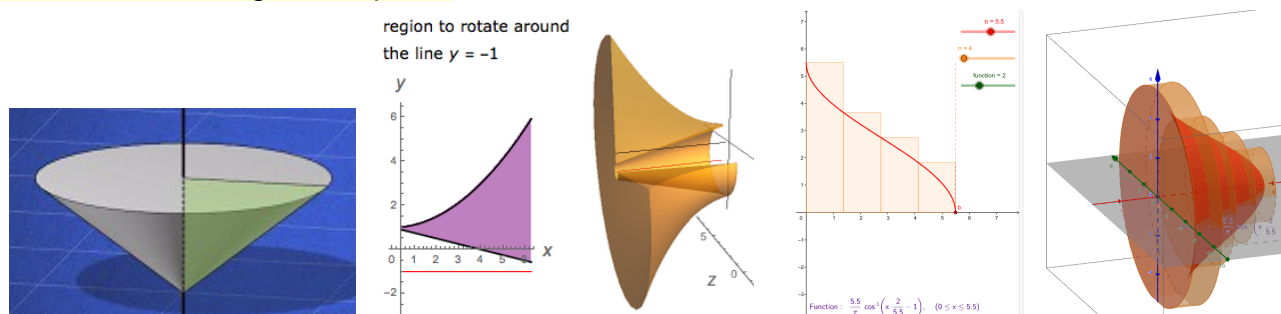
Images by MathBang, Wolfram, Evy Salcedo and Claudio2111 on GeoGebra

Let's model your time-lapse vision of a shape rotating in space. Think of these ornaments as mini-books with pages. You can make these models out of recycled books. Or staple or glue together a mini-book first, and then cut its edge into a shape. Or make a mini-book out of pre-cut, mirror-symmetric shapes.

Images: Rokolee.com, Matt Haney on Instructables, RubbishPlease.com, KrokoTak.com

Experiment with different 2D shapes to see what bodies of revolution they make. Go on a scavenger hunt to see shapes of revolution everywhere: ice cream cones, balloons, car wheels, tree trunks, vases, etc.

MATH
IS WHAT YOU MAKE IT!

►►► **Name** _ _ _ _ _ _ _ _ _ _ _ _ ◄◄◄

~ Examples ~ Questions ~ Problems ~ Stories ~ Conjectures ~ Models ~ Art ~ Formulas ~ Graphs ~

►►► **Research Journal** ◄◄◄

Natural Math

By you and math friends