Combinatorics, Part 2

1 Bike locks, dice, and passwords

1. You are a designer of a combination bike lock. Each combination will consist of 4 digits. For example, one combination is 0-5-8-3.
   (a) How many different combinations are possible?
   (b) What if you only use the digits 1 through 9 instead of 0 through 9?
   (c) How many different combinations are there if you use the digits 0 through 9 and each combination must have at least one 0 in it. Hint: use the answers to part (a) and part (b).
2. You roll a 6-sided die three times. Among all possible outcomes, how many outcomes have one or more occurrence of the number 6? For example, 3-6-1 has an occurrence of the number 6, but 3-4-2 does not. Assume the order of the rolls matters here, so 3-6-1 is different from 3-1-6.

Extra Problem:

3. You need to create a six character password. Each character can be either a lower case letter or one of the special characters #, *, $, %. A password must contain at least one letter and at least one special character. How many different passwords are possible? (Write out how you would find the number, but you don't have to do the arithmetic.)

2 After School

1. You stop by the ice cream shop on the way home from school.

   (a) There are 10 flavors of ice cream available. How many ways are there to make a double scoop ice cream cone with two different flavors, where the order of the flavors matters?

   (b) How many ways are there to make a milkshake with one scoop each of two different flavors of ice cream?

   (c) *What if repetition of flavors is allowed in parts (a) and (b).

2. When you get home after the ice cream, you are greeted by your 4 dogs (Rover, Spot, Daisy, and Killer). In how many ways can you choose one or more of them to accompany you on a walk around the park?

3 Clubs

(a) There are 5 kids in cooking club. In how many different orders can they line up to wash their hands before cooking?

(b) There are 20 kids in robotics club. How many ways are there to choose a president, vice president, and treasurer?

(c) There are 12 kids in Model UN club. How many ways are there to pick a delegation of 3 to represent Russia?

Extra problem

(d) If there are 3 people in a room, and everyone shakes everyone else's hand, how many handshakes take place? What if there are 4 people? 5 people? 15 people?

4 Books and Words

(a) There are 5 books on a shelf. How many ways are there to arrange three of them in a stack on the floor?

(b) How many ways are there to arrange the letters in the word "PART"? For example, one arrangement is APRT. (How many of these form a real word in the English language?!)

(c) How many ways are there to arrange the letters in the word "SWEET"?

(d) How many ways are there to arrange the letters in the word "CHEESE"?

Extra problem:

(e) Dad has two apples, three pears, and four oranges. He will pick one piece of fruit in his son's lunch box for 9 consecutive days. How many ways are there to do this?



5 Numbers and Letters

(a) How many 3 digit numbers are there that have all odd numbers as digits? For example, 337 counts, but 352 does not.

(b) How many 3 digit numbers are there that have at least one even digit? For example, 656 counts, but 137 does not.

(c) How many 3 digit numbers have an even sum of their digits? For example, 125 counts, but 227 does not.

(d) A school has 677 students. Explain why at least 2 students must have the same pair of initials (first and last initial).