Mathematical Induction

March 2, 2024

1. A *bent triomino* is a 2×2 board from which one corner has been removed.

   Is it possible to tile a 16×16 chessboard, from which one corner square has been removed, with bent triominos?

   Tiling a shape means to cover it with tiles so that there are no gaps or overlaps.

2. Prove that the number $111\ldots11$ (with 243 ones) is divisible by 243.

3. In the Tower of Hanoi, 9 rings are arranged on the middle of 3 pegs, in order of their size with the largest on the bottom. On each move, you are allowed to move the top ring from any peg onto the top of the stack on another peg, as long as you don't put a larger ring on top of a smaller ring. Prove that:

   - It is possible to move all the rings to the rightmost peg.
   - You can do so using $2^9 - 1$ moves (assuming 9 rings).

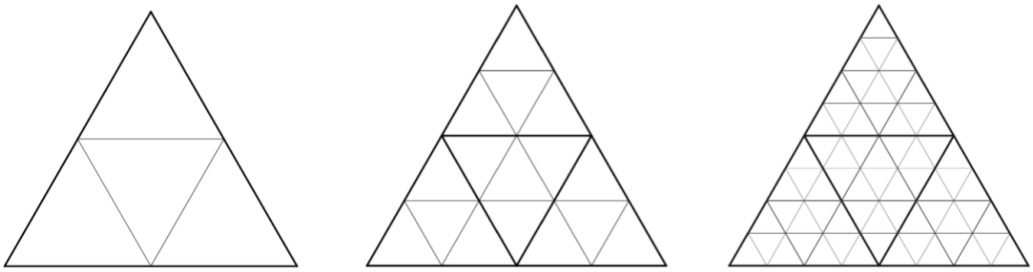
4. What is the maximum number of parts that you can divide the plane into with 7 straight lines?

5. In a warehouse N containers marked 1 through N are arranged in two piles. A forklift can take several containers from the top of one pile and place them on the top of the other pile. Prove that all the containers can be arranged in one pile in increasing order of their numbers with $2N - 1$ such operations of the forklift.

Most of these problems are from *Mathematical Circles (Russian Experience) by Fomin, Genkin, and Itenberg.*

More Tiling Problems

6. A triangular board is subdivided into smaller triangles as shown, so that it has 8 small triangles along each side. A triangular triomino is a *tile* consisting of three adjacent triangles. 

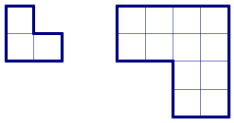


   (a) How many small triangles are inside the large triangle?

   (b) Is it possible to tile the remaining board with triangular triominos after one corner triangle is removed?

   (c) Is it possible to tile the remaining board with triangular triominos after any (one) edge triangle is removed?

7. *A *size-n bent triomino* is the shape you get when you remove one $n \times n$ square from a $2n \times 2n$ square. In the figure below, a size-2 triomino is on the right.



   Can a size-5 bent triomino be tiled by size-1 bent triominos? Can a size-2024 bent triomino be tiled by size-1 bent triominos?