



Mathematics Institute of the Triangle

Session 12, March 16, 2024; materials needed: compass & straight edge

Mr. Barman, March 15, 2024

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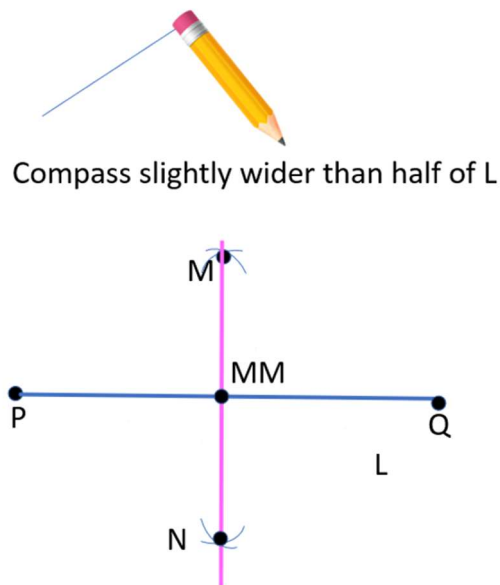
For Chapel Hill Math Circle (grades 1-3)

Happy Pi Day! Let's do some constructions about circles using a compass and a straight edge.

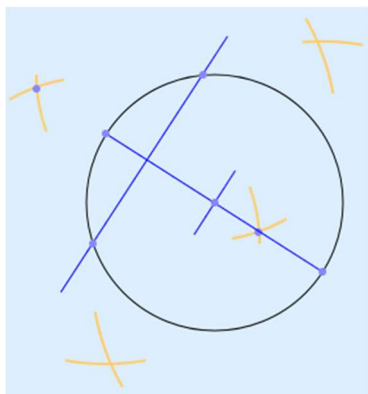
Draw a circle given its diameter

A diameter is like a circle's belt; it goes around its middle. A chord is any line segment that starts and ends on a circle. A diameter is a longest chord.

1. Draw a straight line segment PQ that will be a diameter of the circle.
2. Make your compass a little bit wider than half of that segment's length. Make a little above and below the line with the needle of the compass on one end of the line, then do the same from the other end. Where the arcs meet label "M" above and "N" below.
3. Connect M and N; you have cut PQ in half
4. Where this last line crosses PQ is the center of the circle. Set the compass' needle on the center and extend the compass' pencil to P or Q and then draw the circle.



Find the center of a circle



We will find the exact center of a circle. Of course when you make your circle, you can bear down on the compass' needle and you will then have marked the center. Let's do it another way.

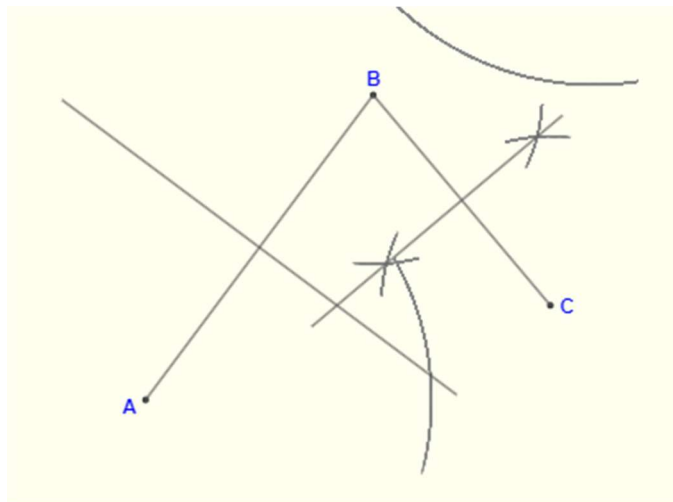
1. Draw any chord through the circle and cut it in half ("bisect it") as above. Draw a diameter by drawing this line from one edge of the circle to another.
2. Now bisect this diameter – do you remember how? Make your compass a little more wide than half the diameter and make arcs on two sides from each end; connect the two intersecting points. Really you just need to draw a little bit of this line, also a diameter, around the first diameter.
3. Where the two lines meet is the center of the circle. This picture¹ may be helpful.

¹ mathsisfun.com/geometry/construct-circlecenter.html

Draw a circle through 3 points

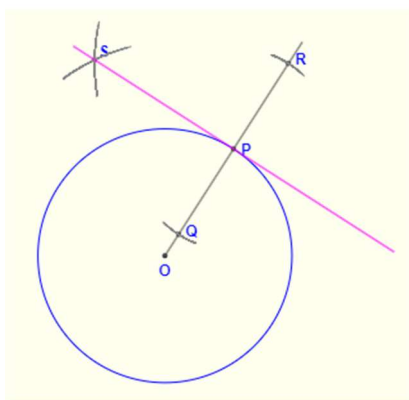
Did you know that if you draw (any?) 3 points, you can find a circle through those points. Well ... almost any. You have to be sure that the 3 points are not all on a line (we say they must not be collinear). If they were then the circle would have an infinite radius!

This diagram from Math Open Reference² may help.



1. Draw 3 points and label them A, B, and C.
2. Pick any pair – maybe A and B. Pretend there is a line segment between them (in the picture they actually drew one but I recommend not as it makes the construction busy). Bisect this. I bet you remember how! Make your compass a little wider than half of AB, make little arcs on either side from each of A and B, and connect where the arcs intersect.
3. Repeat with another pair – maybe A and C (here they made the other remaining choice, B and C).
4. The point where the two bisectors meet is the center of your circle!
5. Put the needle of your compass on this center and extend your compass to any of the three points A, B, or C. Draw your circle!

Can you find a line just touches a circle at a point (the circle's “tangent” at that point”)?



I bet you can figure it out; this diagram from Math Open Reference³ can help. Start with a circle and its center, plus a point (P here) on the circle.

1. Draw a line from the center to P and beyond.
2. Set your compass to be less than P to the center; put your compass' needle on P and make little arcs on either side. Label where these arcs intersect the line Q and R.
3. Bisect the imagined line QR - you know how to do that! Make arcs from Q and R on either side (one side is missing here) with the compass a little wider than QP. Connect the two intersections - you've made a tangent line!

Aren't constructions fun? See if you can figure out how to make an equilateral triangle. Enjoy!

² mathopenref.com/const3pointcircle.html

³ mathopenref.com/consttangent.html

Have Fun!
Mr. Barman