Natural Math

# Circles and Triangles IN SPACE!

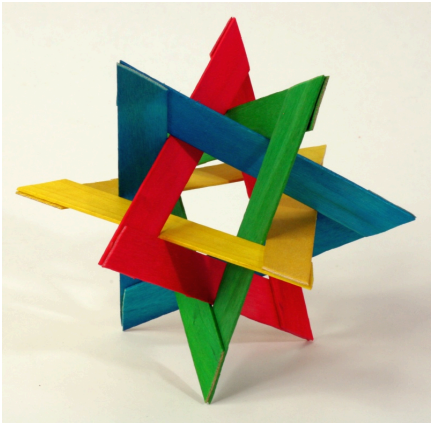
## PATHWAYS

**#51 Geometry**
**#52 Convex and Discrete Geometry**

## Family Math Circle ~~~ February 2024

**Pathways name our math inspirations using Mathematics Subject Classification. Mathematicians around the world label their work with library codes from MSC, https://msc2020.org/ Let us help students feel happy familiarity with each subject area!**

**Family Math Circle is an informal learning space where participants make advanced mathematics accessible to everyone in kind ways.**



*4-triangle polylink by Making Math Visible*

## What Will Everyone Make?

**Roots and wings: Make models. Make connections.**

Construct circles and equilateral triangles in different ways to explore 2D geometry and trigonometry in Flatland. Fold origami to make polylinks and other sculptures out of triangles to study 3D geometry in space.


## Words With Math Friends

**Tell friends and family all about your math creations. Use these terms +"math" to find images, videos, and articles on the web.**

- Triangle, side, vertex, angle
- Equilateral triangle, side
- Circle, center, circumference, arc
- Origami, fold, pinch, crease
- Pyramid, tetrahedron
- Polygon, polyhedron, net
- Dimensions, 2D, 3D
- Polylink

*Add your family's own words to the Research Journal. Use these personal terms along with the standard terms.*

## Interesting Choices

**Mathematicians do many different things. What kind of math person will you be today?**

How will you make a circle? Will you make equilateral triangles by drawing, folding circles, or assembling sticks? Do you like tiny sculptures, tabletop sculptures, or giant constructions?

*These are starter choices. You will come up with more ideas. Add them to the Research Journal.*

## Toolbox

Physical (gray=optional): Paper, colored pencils, scissors, tape or glue, compass, protractor, round objects to trace, cardboard-like materials.

Virtual:
From tetrahedron to polylink
https://youtu.be/ImzpnIwiWUQ
MakingMathVisible.com

https://naturalmath.com/circles/

Extra activities, videos, math connections, books, and other resources for math circle leaders.

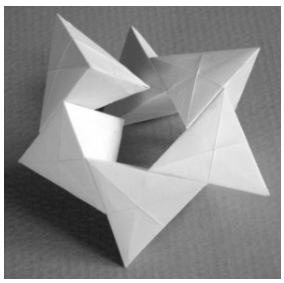


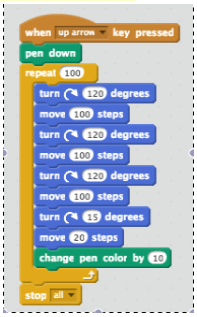
*Photo: WholeMovement.com*

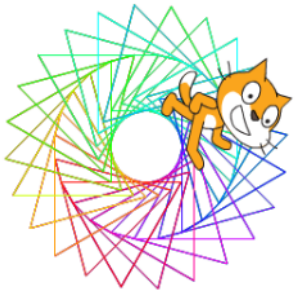
# Circles and Triangles IN SPACE!

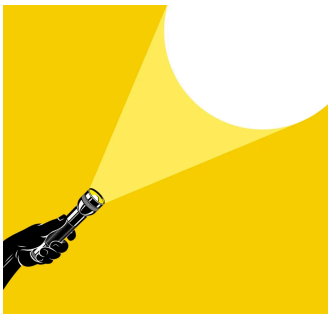
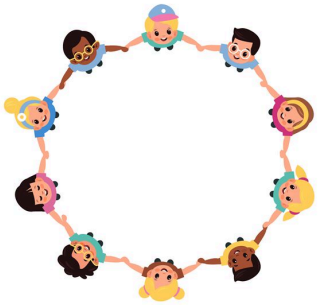
## 0. Warm-up: How Do You Make Circles? What IS a circle?

How do you make a circle? Over the millennia, people all over the world came up with many inventive ways! You can use paperclips or rope, software like Scratch or Geogebra, a group of friends holding hands, or your body leaving traces in the sand. Think of 4-5 ways you could make a good circle. It won't be perfect, but that's okay! All models are wrong but some models are useful. They help us imagine the perfect mathematics in our minds. What IS a circle, anyway? Pretend your math friend is a space alien from a no-circle world, and explain circles in your own words.









*Images: Kelly Medford and Ali Garbacz on WikiHow; 14th century translation of Euclid's elements; Scratch from MIT; CraftifyMyLove; YummyBuum*

SPOILER

To make circles from scratch, think about moving. How? (1) Compass: Keep a constant distance from the center with a tool. (2) Copy: Trace or project a circle you have. (3) Stretch-a-loop: Stretch a looped band into many directions with equal force. (4) Vehicle: Fix the wheel at an angle and drive at a constant speed. (4a) Digital vehicle: Make equal steps and turns, on repeat. (5) Calculus: Assemble or cut out many equal-length, thin objects around the center.

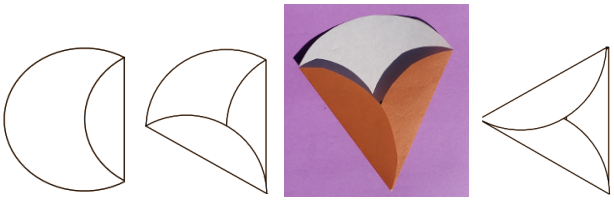
## 1. Find the Center of Your Circle

Draw a circle on paper. Choose the size: 1 to 3 widths of your hand. Tiny or huge paper circles are annoying to handle. Cut out your circle. How can you find the center of your circle? Think of 2-3 ways.

Here is one way. Start folding your circle in half. Lightly pinch around the middle and unfold. Start folding your circle in half another way and pinch in the middle again. Pinched circles make pretty curved surfaces in space! The two small creases in the middle will cross at the center. Draw a dot there.

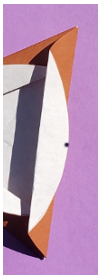
## 3. Fold an Equilateral Triangle

Fold the edge of your circle (the circumference) to the center. It will make a nice arc. Trace the crease line with your finger. From one endpoint of that crease line, make another fold that puts your circle's edge onto its center. Your two arcs will make a shape that resembles an ice cream cone. Fold the third arc to the center.



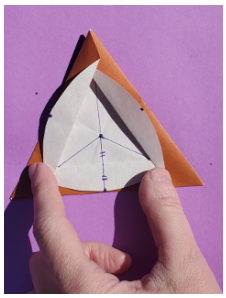
Unfold your circle and take a look at the special triangle you constructed inside: an equilateral triangle. What do you notice about your equilateral triangle?

## 4. Doubles, Halves, and Half-Way Triangles

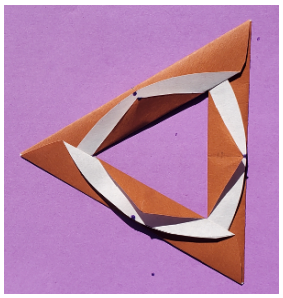


Mark the midpoints on the three arcs that you folded earlier and the three sides of your triangle. How do you find those midpoints? Fold point to point on all three sides. You will reveal a smaller triangle inside of your larger triangle. Unfold, refold, and play with your sculpture. What do you notice about the sizes of parts?


There is only one, unique way to fold a point onto another point. This is the second of the seven axioms of paper-folding, called Huzita-Hatori axioms after mathematicians who published their work in 1990s.
https://en.wikipedia.org/wiki/Huzita%E2%80%93Hatori_axioms#Axiom_2

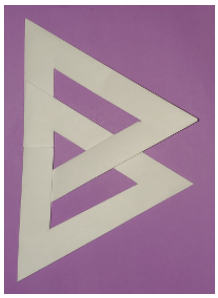


By now, there are lots of halves, midpoints, and double lengths in your folded sculpture! Go on a scavenger hunt to find distances twice as long as other distances. How many such pairs can you find? That 2:1 ratio appears whenever you see the special angles (60-degree angles) from these special equilateral triangles. Mathematics of triangles is called trigonometry.



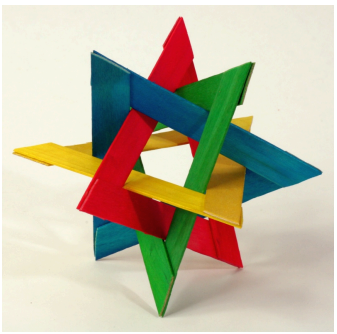
After exploring, glue down your construction to make it flat and sturdy. Fold or draw lines connecting the center and the vertices of your small triangle. Cut along those lines, fold the flaps over the arcs, and glue them down.

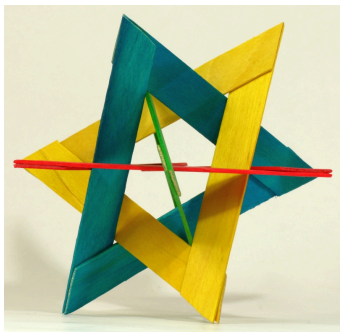
## 5. The Four-Triangle Polylink



Make another holey triangle as you did before, in another color but the same size. Cut your second triangle across one of the sides, half-way between the midpoint and the vertex. Link the two triangles together. Because of the many 1:2 ratios in your triangles, they fit one another in many pleasing ways. Play with the two linked triangles and arrange them into your own designs. What do you notice? After playing, close up the cut on the side of your second triangle with tape or glue a piece of paper over it.

The following puzzle IN SPACE is easier with friends who hold triangles in place for you. Link a third and then a fourth triangle together into a sculpture called polylink. Watch photos, videos, or ask experienced math friends to help arranging your polylink. Many mathematicians decorate their rooms with sculptures like these. You can 3D-print the triangles or use any long, thin objects to assemble them side by side.





*Images: Making Math Visible and Vi Hart*

IS WHAT YOU MAKE IT!

▸▸▸ Name _ _ _ _ _ _ _ _ _ _ _ _ _ _ ◂◂◂

Words ~ Examples ~ Questions ~ Problems ~ Stories ~ Conjectures ~ Models ~ Art ~ Formulas ~ Graphs ~

▸▸▸ Research Journal ◂◂◂