

Dihedral Groups and Snowflakes

PATHWAYS

#13 Commutative Algebra

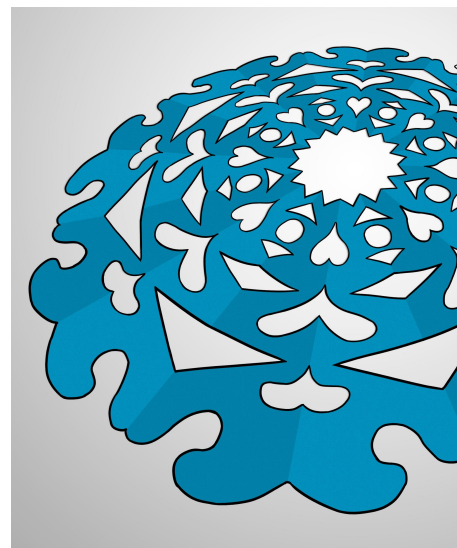
#20 Group Theory and Generalizations

#51 Geometry

Family Math Circle ~~~ November 2023

Pathways name our math inspirations using Mathematics Subject Classification. Mathematicians around the world label their work with library codes from MSC, <https://msc2020.org/> Let us help students feel happy familiarity with each subject area!

Family Math Circle is an informal learning space where participants make advanced mathematics accessible to everyone in kind ways.



From "Moebius Noodles Adventurous Math for the Playground Crowd"

Roots and Wings

Make models to ground your math. Make connections to advance in higher math.

Make and find examples of polyhedra to explore the geometry of polygons in 3D space. Fold and cut paper to create snowflakes and other symmetric art and connect it to algebra and group theory.



Words With Math Friends

Tell friends and family all about your math creations. Use these terms + "math" to find images, videos, and articles on the web.

- fold, edge, hole, notch, paper model
- face, edge, vertex, angle, polygon, polyhedron (plural: polyhedra), nets for polyhedra
- to reflect, line symmetry, to rotate, radial symmetry
- group, group of symmetries, dihedral group, subgroup

Interesting Choices

Mathematicians do many different things. What kind of math person will you be today?

Go on a scavenger hunt for shapes based on their properties, or make your own models with the properties you desire. Choose how to fold, how many times, where to cut, and in what shape to create unique snowflake paper models of groups of symmetries.

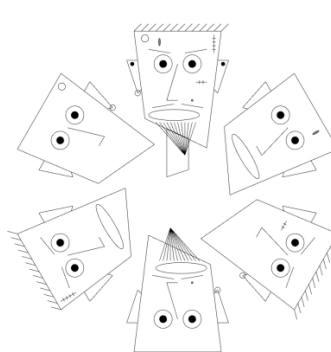
Toolbox

Physical (gray=optional)

Paper, colored pencils, scissors, tape, mirrors, mirror books, paper punch.

Virtual

Symmetry painting <http://weavesilk.com/>
Wallpaper groups explorer <https://eschersket.ch/>
Symmetries with GeoGebra
<http://www.malinc.se/math/geogebra/symmetriesen.php>



NaturalMath.com/circles/

Extra activities, videos, math connections, books, and other resources for math circle leaders.

David A. Reimann, "Visualizing Symmetry Subgroup Structures Using Simple Motifs," Bridges Math Art conference 2018

Dihedral Groups and Snowflakes

0. Warm-up: A Sheet of Paper IN SPACE!



Make or find some polyhedra: cubes, prisms, pyramids, and other objects made from flat polygons. What are polygons? Find or draw some polygons of your own: flat shapes with straight edges. When polygons come together to make a shape in space (a 3D shape), they become faces of that shape. Draw smileys or animal faces on your polyhedra and count them. Can you imagine, find, or make shapes with 6 faces? (There are a lot of those around!) How about 8 faces (octahedron) or 4 faces (tetrahedron)? Can you see a common sheet of paper as a shape in space, and its front and back as two faces ("a dihedral

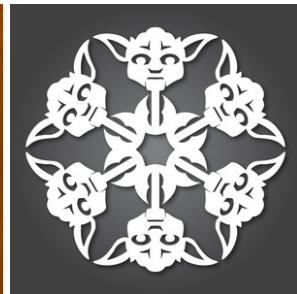
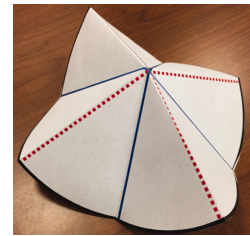
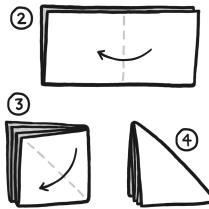
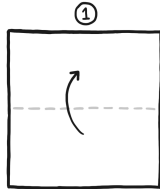
shape")?

Image: HeartAndMindPress.com



You could experiment with paper shapes and figure out how to make 3D shapes out of polygons in your own ways. Many people enjoy that as an endless series of puzzles. Math people often share their design for making polyhedra, called nets. If you search the web for printable cube nets, printable prism nets, or 3D shape nets in general, you can print, cut, and fold someone else's design.

1. The Secret of Snowflake Crease Patterns



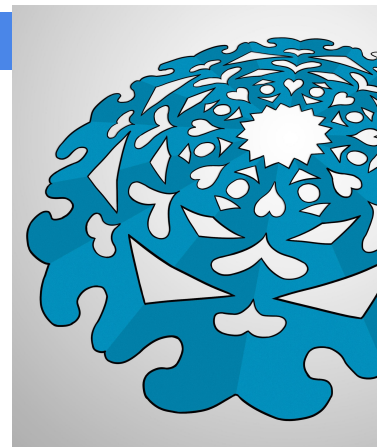
Images: Lily Hong Lei, Yreina Florez Orte, Anthony Herrera, Nicholas Turner et al.

People around the world make round papercrafts similar to snowflakes. There are lots of ways to do that! Experiment with folding paper in different ways, then unfold and admire your crease patterns. Can you find ways to fold paper where every single crease line goes through one point? That's called a snowflake fold.

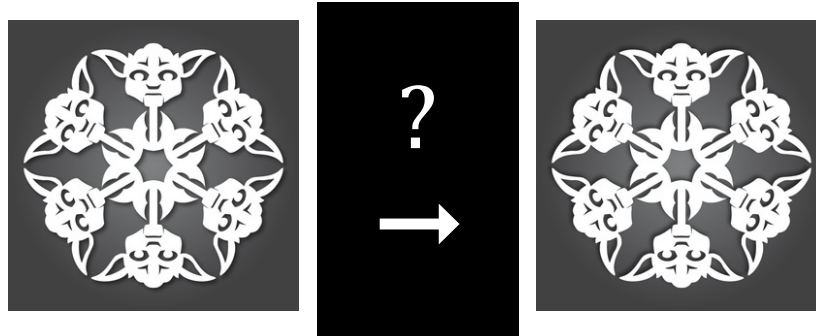
3. Fold, Fold, Cut

Fold paper into a snowflake fold. Cut out notches at the edges and corners of your paper. Unfold and admire your unique paper snowflake! Try different math experiments:

- * Cut to make holes shaped like hearts, diamonds, or other cute things.
- * The shapes you cut out multiply! How many times? Can you change that?
- * Make weird and wild cuts and see what surprising shapes those make.
- * How can you make a star in the middle?



4. The Snowflake Secret Strikes Back



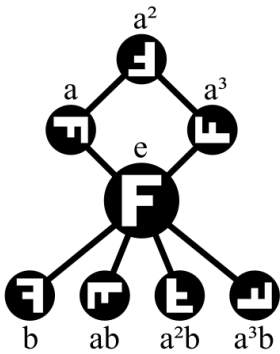
In the picture above, we played with the Yoda snowflake IN SPACE. Maybe we reflected it (flipped it to the other side), or rotated it around, or some combination of these moves. Yes, the result looks exactly the same! But we moved the snowflake, promise!

Move your own snowflake in a secret way that doesn't show that you moved it at all. Ask your math friends to guess what you did. How many different secret moves like that can you find?



This has to do with symmetry!

4. The Guardian of the Group



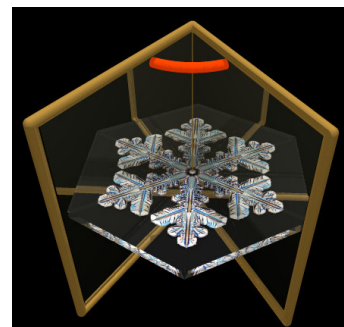
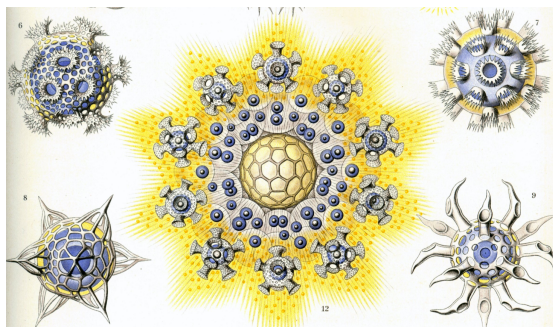
Want to better see and count the secret moves of snowflakes? Add a sticker to your snowflake, with a special "guardian" character that is not symmetric in any way. Your character can be a creature like a dog, a letter, or a number. Math people traditionally use the letter F.

Together, the results of all the possible secret moves of the snowflake (where the results of each move look the same as the original) are its symmetry group. You will notice that some moves from the group take your special character there and back again. For example, you can flip the snowflake to its other side (reflect) and then flip

it back (reflect once more). Results of the moves like that form a subgroup. Look for subgroups of your snowflake's group of symmetric moves.

5. Make and Find Your Own Symmetry Groups

- Make all kinds of paper snowflakes! Big ones, small ones, with lots of cuts, or just a few.
- Learn to make paper snowflakes with 3, 4, 5, 6, 7, etc. points.
- Find other objects that have snowflake-like symmetry and therefore form the same symmetry groups. There are a lot of them in nature and culture.



Images: Ernest Haeckel; Ester Dalvit



»»» Name _____ «««

~ Examples ~ Questions ~ Problems ~ Stories ~ Conjectures ~ Models ~ Art ~ Formulas ~ Graphs ~



»»» Research Journal «««

