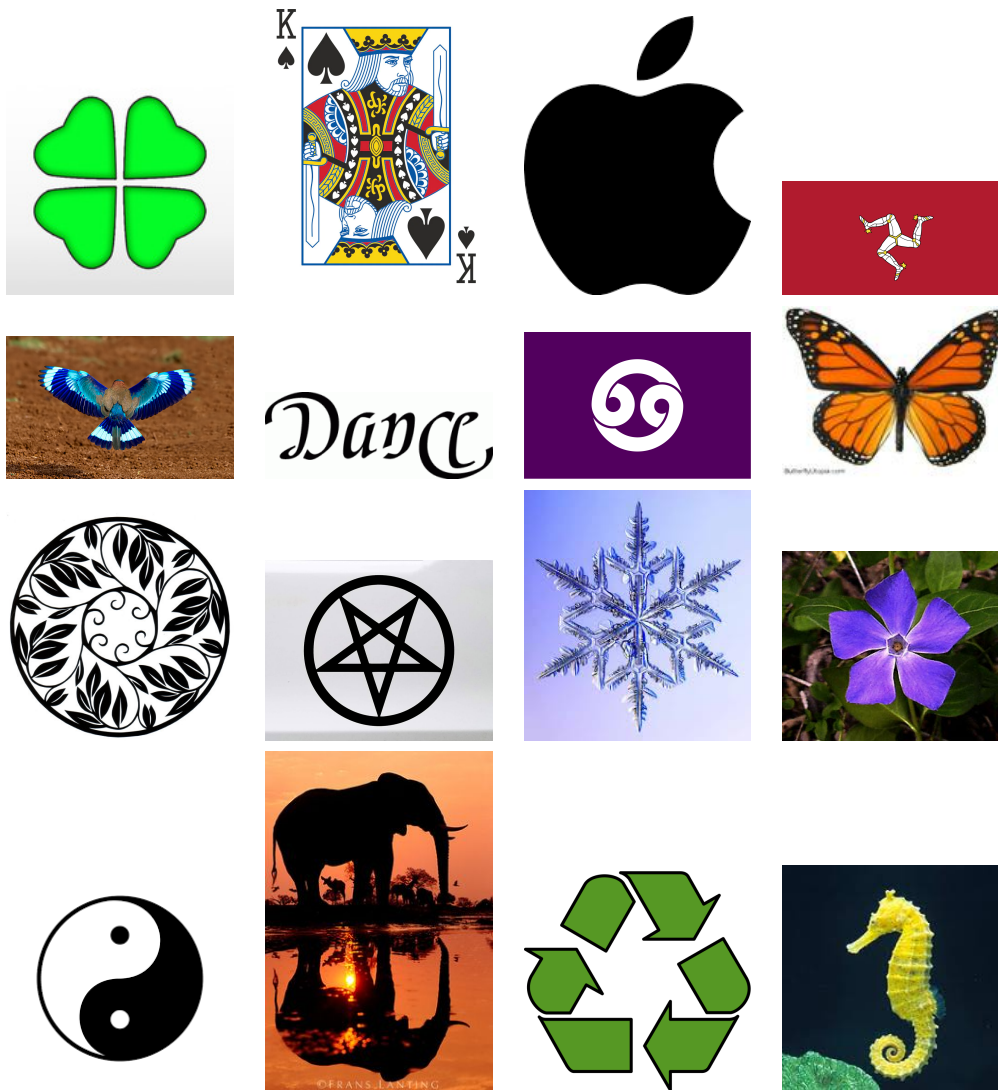


# Envelope Tilings <sup>1</sup>

## 1 Reflections and Rotations

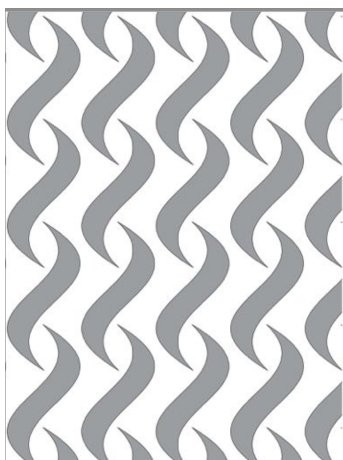
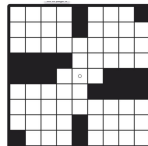
1. Help me sort these shapes into four piles, and guess the rule I am using to sort them.



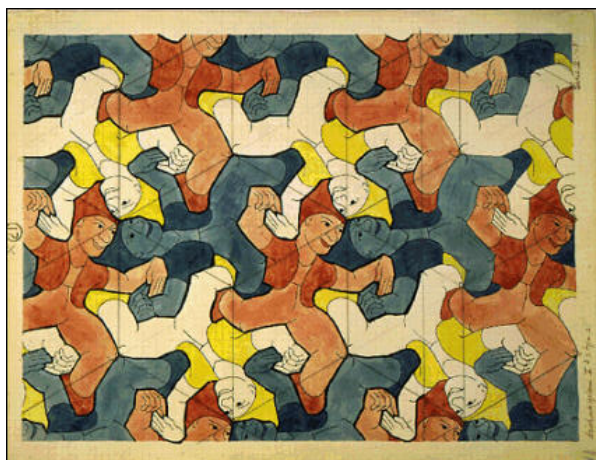
<sup>1</sup>From Chaim Goodman-Strauss's TootiTooti lesson plan.

## 2 Rotational Symmetry

2. Find the rotation points in each figure.

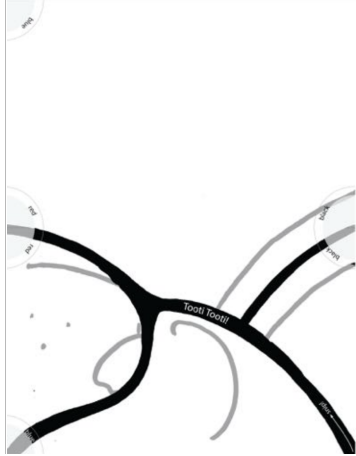


3. How many different *types* of rotation points are in each figure? Ignore color.



### 3 Envelope Tiling

4. Take a piece of paper that looks like this:



- (a) Color the points marked red, blue, black
  - (b) Fold the paper in half hamburger style and tape the edges together to make an “envelope”
  - (c) Cut along the dark black lines. Be careful not to cut both sides of the envelope.
  - (d) Unfold your shape
5. Try to fit your piece together with other pieces that other students have cut out to make a tiling.
6. How many different types of rotation points does the tiling have? What points on the envelope did they come from?

### 4 Envelope Tiling DIY

7. Now make your own tiling!
- (a) Take an envelope, and make sure the flap is sealed, or make an “envelope” by folding and taping a sheet of blank paper
  - (b) Draw some lines or curves on the envelope. What properties should these lines have?
  - (c) Cut out on the lines. Be careful not to cut through both sides of the envelope! It is fine to snip off one corner of the envelope to get started.
  - (d) Unfold your shape and trace it on a whiteboard
  - (e) Trace lots of copies that fit together
  - (f) What does your shape look like?

## 5 Folding up Orbifolds

We have cut envelopes into tilings. Can we also fold tilings into envelopes?

8. Take one of these two tilings.



- Draw some of the rotation points. Be sure to find all the different types of rotation points. How many are there? If possible, mark different types of rotation points in different colors.
- Connect the points with lines to make triangles or parallelograms
- How many of these triangles or parallelograms do you need to so that every part of the pattern is somewhere within them?
- Cut out that many triangles or parallelograms and tape together sides where the patterns match up. This is your “envelope”, which is the *orbifold* of the tiling pattern.

