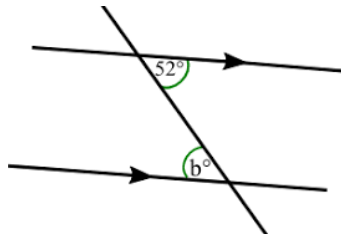
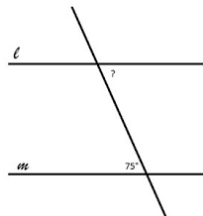
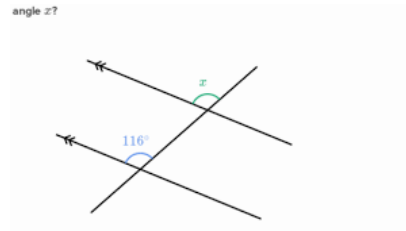
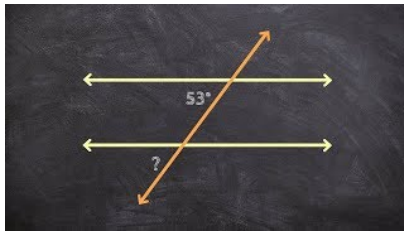
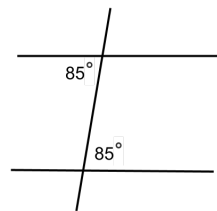
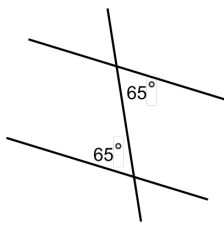
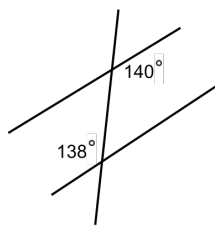


Tiling ¹

- What is the sum of all the angles in:
 - a triangle?
 - a rectangle?
 - a quadrilateral that is not a rectangle?
 - a pentagon?
 - a hexagon?
- What does it mean for two lines to be parallel?
- What are the missing angles labeled in these figures? Assume that the two lines that look parallel are parallel.



- Based on the angles shown, which figure shows parallel lines?



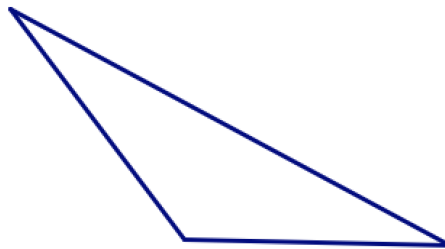
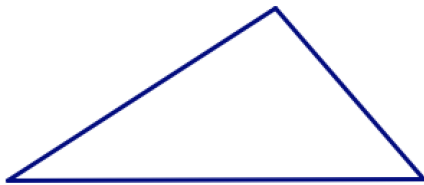
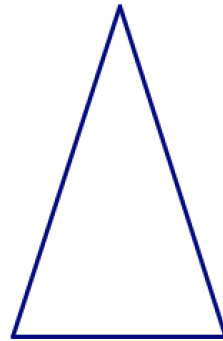
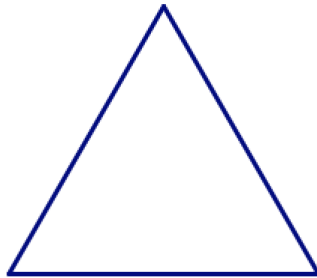
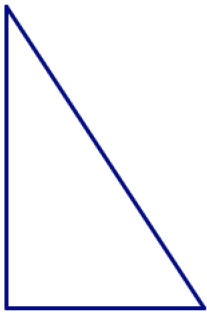
¹Problems from *Geometry: A Guide for Teachers* by Judith and Paul Sally, Mathematical Circles Library Volume 3.

5. A shape *tiles*, or *tessellates* the plane if it is possible to cover the plane with copies of the shape, with no gaps or overlaps. It is okay to reflect the shape (flip it over).
- (a) Does a square tile the plane?
 - (b) Does a circle tile the plane?
 - (c) Does a rectangle tile the plane? Can you find more than one pattern for tiling the plane with rectangles?

1 Tiling with Triangles

Which of these triangles can you tile the plane with?

1. A right triangle?
2. An equilateral triangle?
3. An isosceles triangle?
4. A scalene triangle?

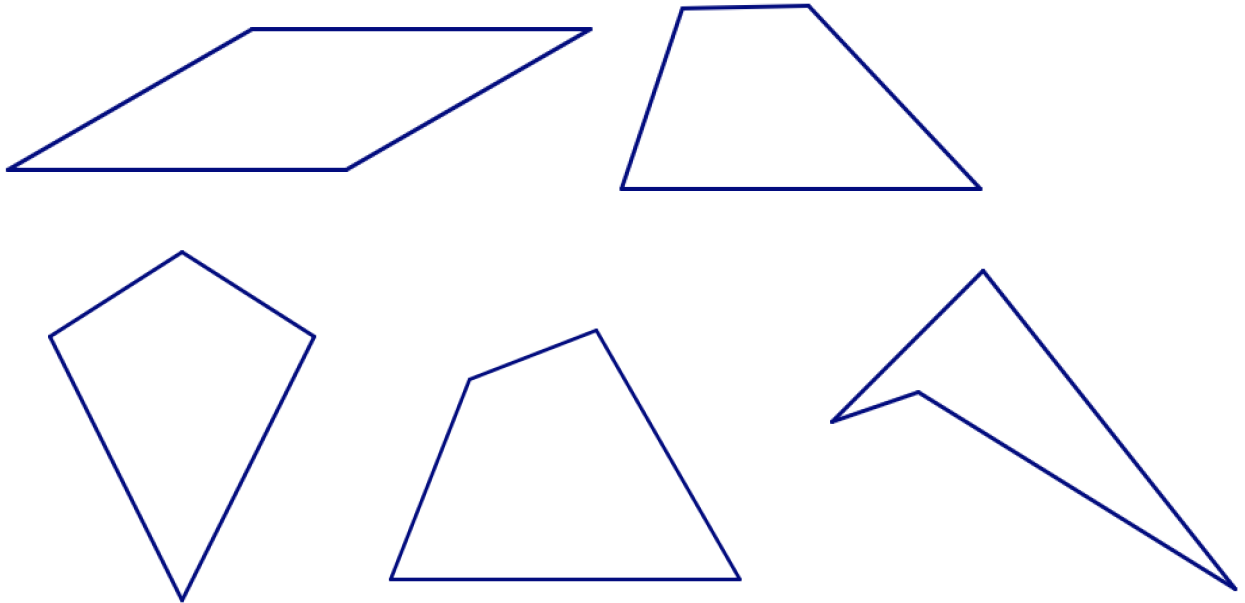


How many *different* ways of tiling the plane can you find with these triangles? (Each tiling should just include one type of triangle.)

2 Tiling with Quadralaterals

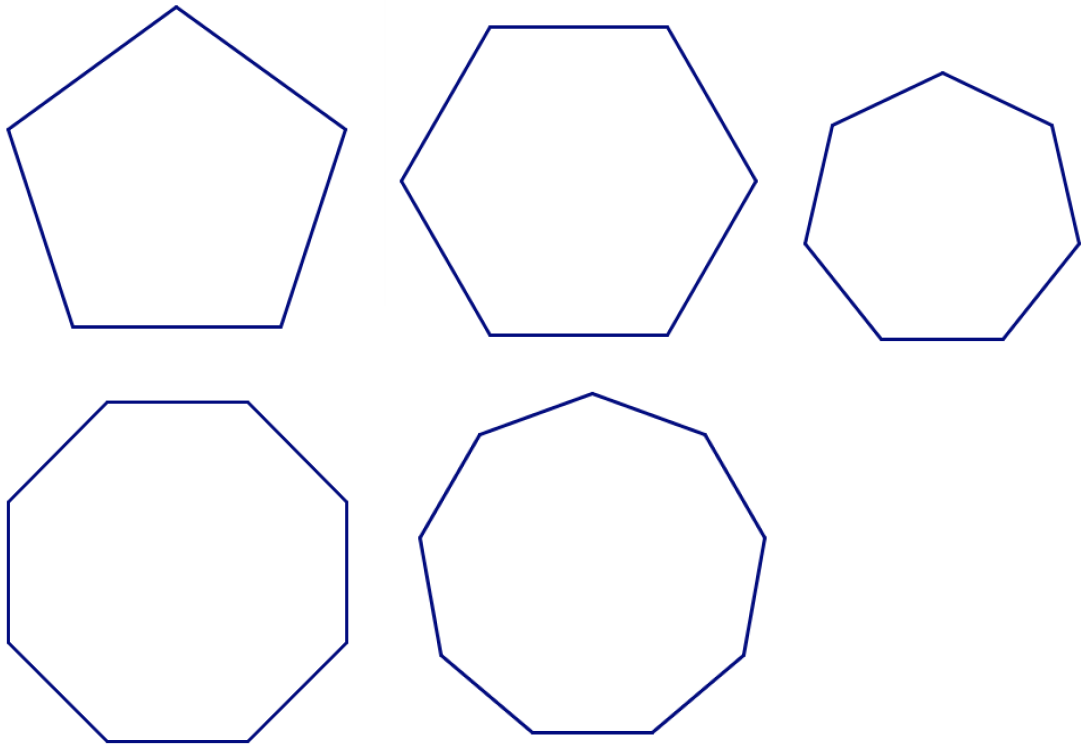
Which of these quadralaterals can you tile the plane with?

1. A parallelogram?
2. A trapezoid?
3. A kite?
4. A *convex* quadralateral that is none of these shapes? (A shape is convex if any line segment between two points on the shape lies within the shape.)
5. A *concave* quadralateral? (A shape is concave if it is not convex.)



3 Tiling with Pentagons, Hexagons, and Other Polygons

1. Can you tile the plane with *regular* pentagons? A regular polygon has all sides the same length and all angles equal.
2. Can you tile the plane with regular hexagons?
3. Can you tile the plane with
 - (a) regular heptagons (a 7 sided polygon)?
 - (b) regular octagons?
 - (c) regular nonagons?



4. Which of the following pentagons and hexagons tile the plane?
5. What other pentagons and hexagons can you draw that will tile the plane?
6. What do you notice about the opposite sides of some of the hexagons and pentagons that tile the plane?
7. Can you tile the plane with any kind of octagon? (It does not have to be convex.)

