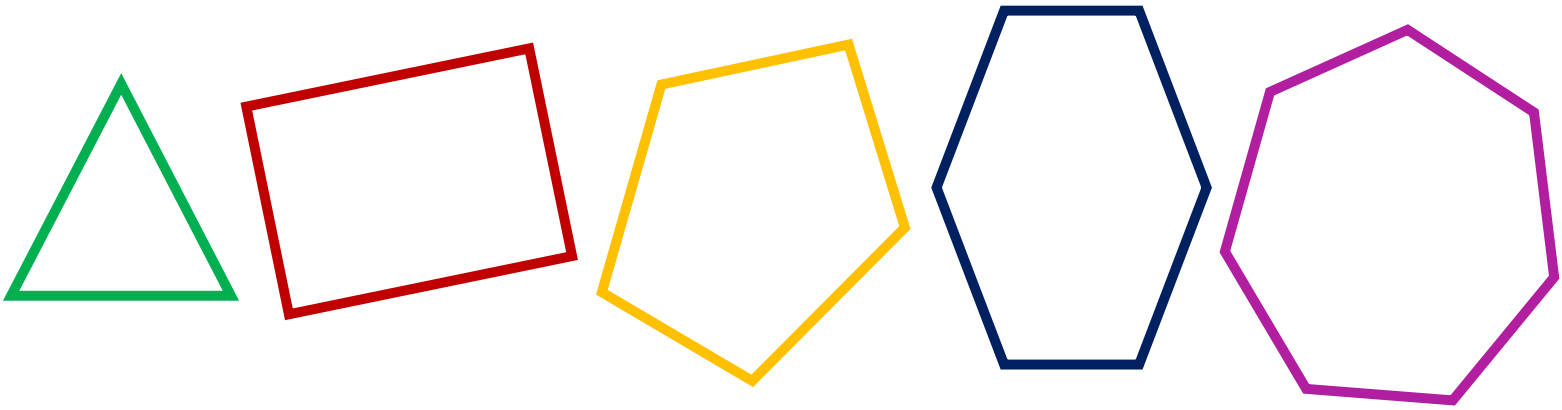
Chapel Hill Math Circle
Session 4 – November 4, 2023: Diagonals, and Pascal's Triangle
Beginners' Group (grades 1-3), 10:30-11:30a
Mr. Barman – dilip@trianglemathinstitute.com
Supplies needed: rulers, spaghetti, paper, colored pencils, optional rope

Welcome to Chapel Hill Math Circle! We're glad that you are here to have some fun! We've been talking about shapes and started discussing diagonals. Let's pick up with diagonals and then talk about a special triangle called Pascal's Triangle.

Diagonals

We decided that a diagonal is a straight line segment between two "vertices" or corners of a polygon that aren't directly connected. We were starting to count diagonals.



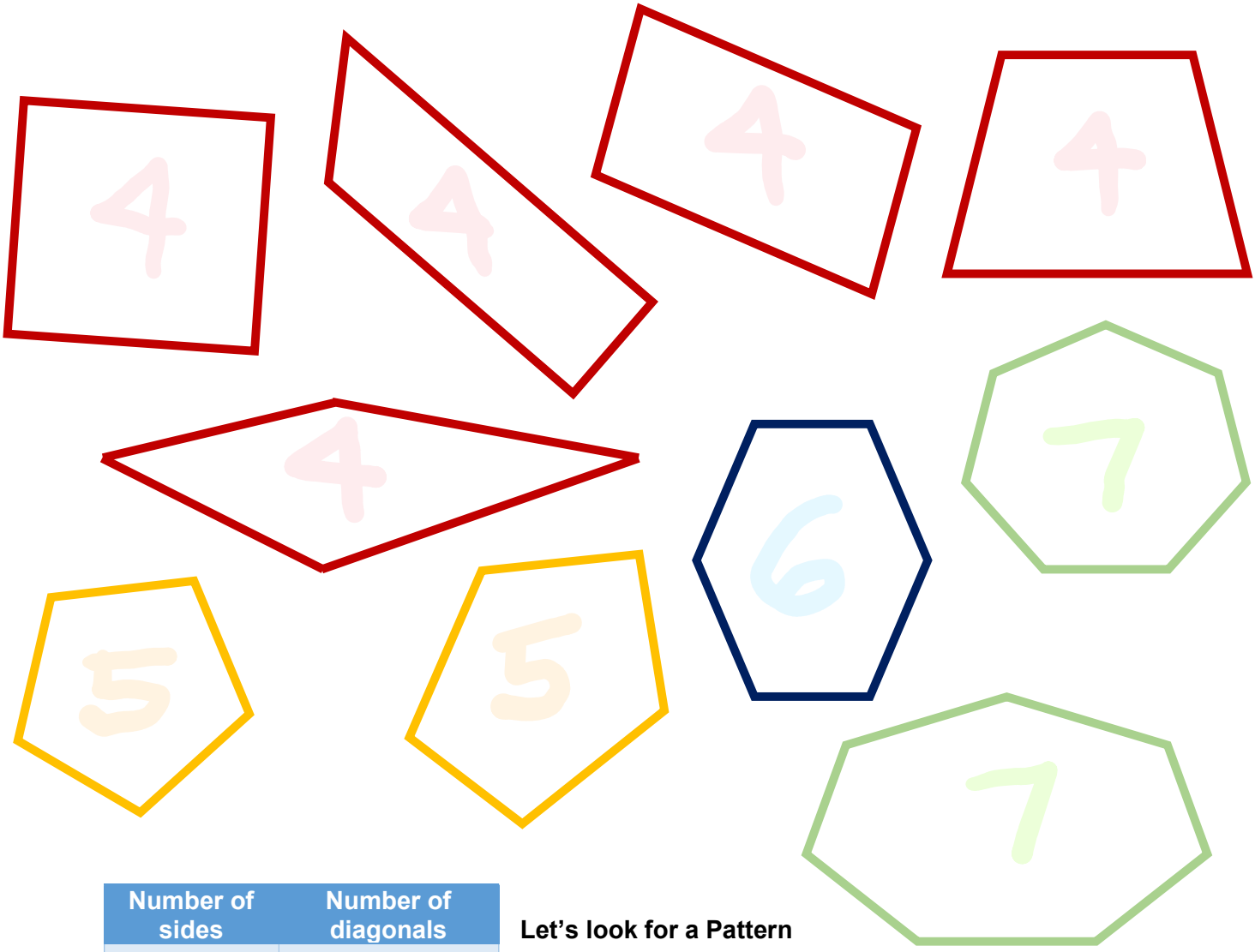
What we learned about diagonals

- A triangle has no diagonals; do you remember why? Let's call a triangle a 3-gon.
- A 4-gon (quadrilateral) has 2 diagonals; do you remember why?
- We decided that all polygons with the same number of sides have the same number of diagonals – all 3-gons have 0 diagonals, and all 4-gons have 2 diagonals. Why?
- We were trying to find a pattern about how many diagonals a shape with a certain number of sides (an "n-gon") has.

Take a few minutes and play with the polygons above and your own polygons to see if you can figure out how many diagonals polygons have. I have more polygons on the next page and have lighty included the number of sides for each.

Let's make a table

Play with the polygons here and/or draw your own. It's easier if you stick with convex polygons. Count the number of diagonals different polygons have and see if you can find a pattern. Can you predict how many diagonals an n-sided polygon would have when n can be any positive integer?



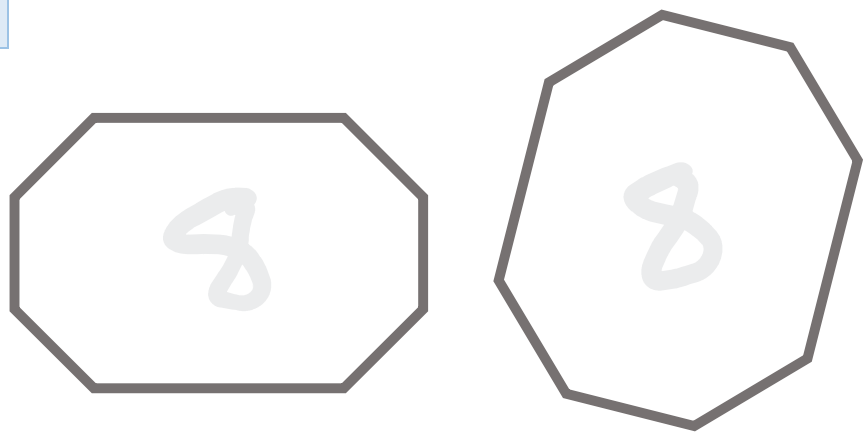
Number of sides	Number of diagonals
3	0
4	2
5	
6	
7	
8	
9	
10	
n	

Let's look for a Pattern

I'm going to play with a pentagon and look at a few ways to count diagonals. Why don't you play first before looking on the next page?

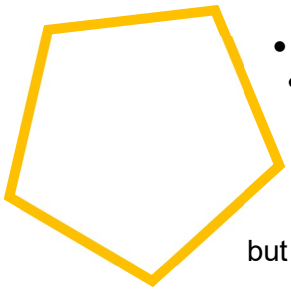
Look for patterns in the pictures that you draw. A 3rd grader last time found a really interesting pattern by looking carefully at how each number differed from the others.

Play for a few minutes before we move on to the next page. Enjoy!

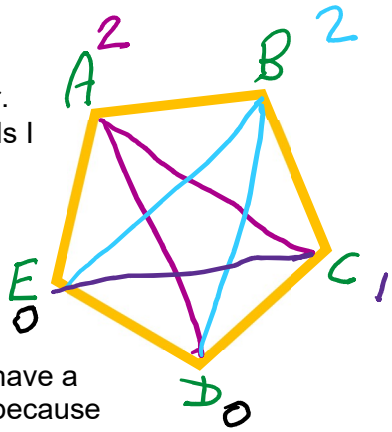


Here's how I am going to play with a pentagon

I'm going to try to count the diagonals but will do it in an orderly way.



- I have an idea – let me label each vertex with a letter.
- I'll then go in order and count the number of diagonals I can have.



- Let's start with A. It can't go to itself or its neighbors but can go to C and D.
- How about B? It can't go to A or C but can go to D and E. We also have a shortcut – we don't have to consider vertices we've already visited because we've already thought about those connections.
- Now we're at C. Ignoring itself and earlier vertices, we consider D (no – neighbor), E (yup), … and that's it.
- At D we look at … nothing! That's the same for E.

So in our pentagon we have 2 + 2 + 1 + 0 + 0 = 5 diagonals.

Here's another way to count them:

- Start with A: AC, AD (2)
- Start with B: *I don't have to look backward as I've counted from A already and don't want to double count, so I start with D (C is my neighbor so there's no diagonal)* BD, BE (2)
- Start with C: CE (1)
- We're done – there can be no diagonals from D (since the first possibility is A, but we've already considered A) and definitely not from E (0) (0)

But this is difficult to generalize – or is it? Can you think of any other way to count that you can generalize? *Hint: ignore double counting and then take half of what you get.*

Your notes on counting diagonals

What you have learned about diagonals

We are going to move on shortly to a new topic, Pascal's Triangle. But before we do take a few minutes to summarize what you have learned and what you have explored.

- What is a diagonal of a polygon?

- Why does a triangle have no diagonals?

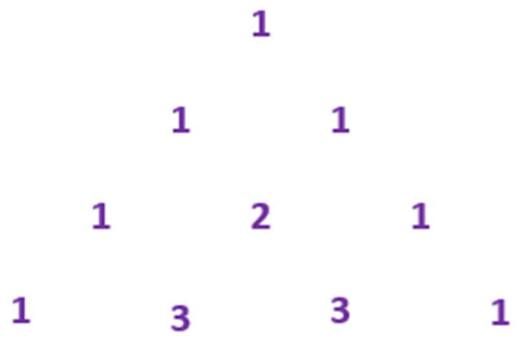
- What patterns have I found in diagonals – can I predict how many diagonals an n-sided polygon will have?

Number of sides	Number of diagonals
3	0
4	2
5	
6	
7	
8	
9	
10	
n	

Number of sides	Number of diagonals	What happens when we get 1 more side?	What happens now row by row?
3	0		
4	2		
5			
16			
7			
8			
9			
10			
n			

Pascal's Triangle

Write down the number 1 near the horizontal center of a piece of paper. On a line below it (the next row), start the row with a 1 and end it with a 1 on either side of the top 1. For the following row, start and end with a 1 but then for the middle value add the two numbers. Keep going; start and end each row with a 1 and find out the other values by adding. You should end up with a triangle that looks like the following after 4 rows:



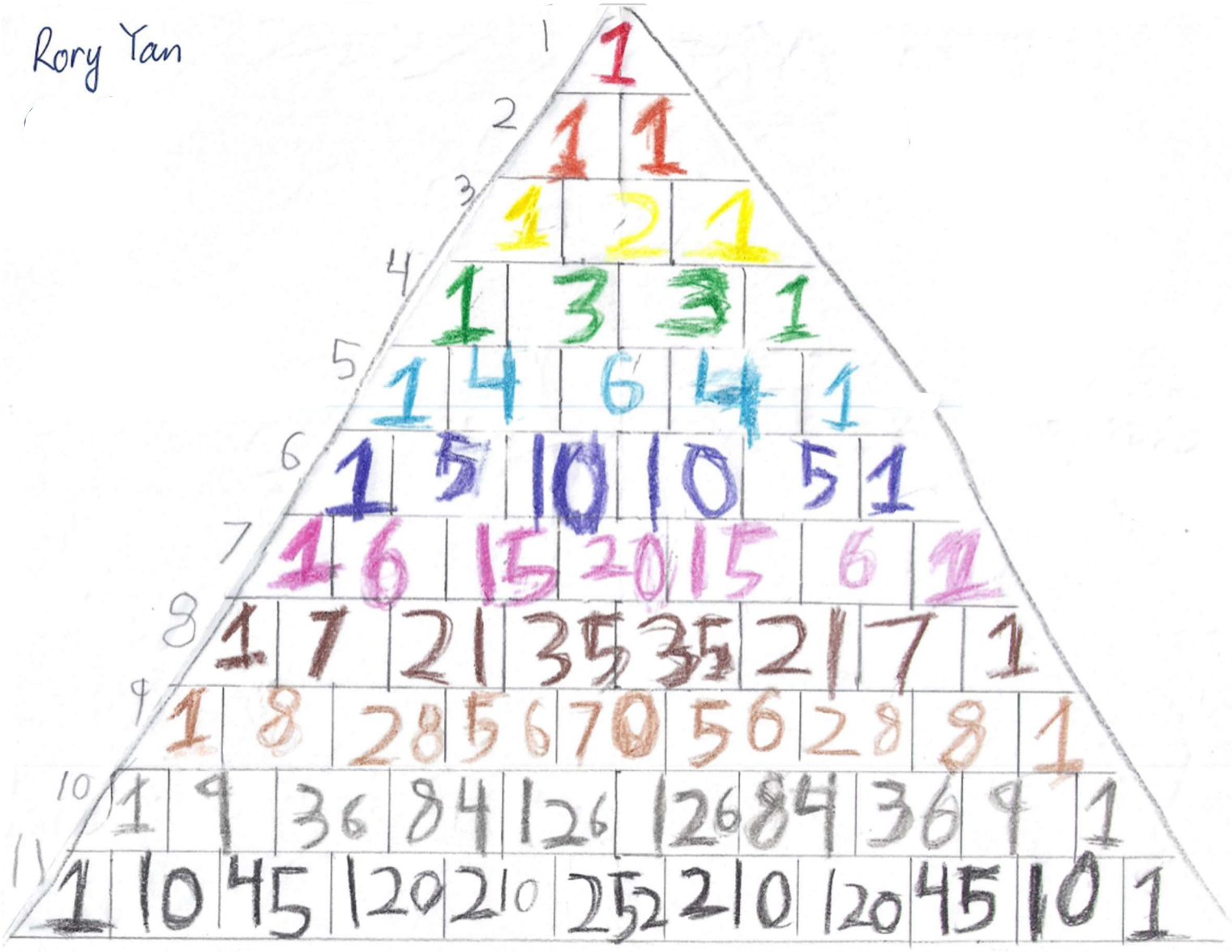
Blaise Pascal

This triangle was popularized in 1653 by Blaise Pascal[1], a remarkable mathematician and philosopher who was brilliant at math as a child. He created one of the first calculators and helped to create important areas of math, economics, and social science. He didn't invent it; it was well known starting over 1000 years earlier in India, Greece, China, and Persia. He knew this and didn't take credit but another mathematician, Raymond de Montmort, named it after Pascal in 1708.

[1] The picture here is from an engraving of Pascal by Henry Hoppner Meyer almost 200 years ago in 1833; I got it from britannica.com/biography/Blaise-Pascal, accessed Jan. 9, 2023.

What patterns do you see in Pascal's Triangle?

Draw one and see what you notice. Do you wonder why these patterns are there? To help you and provide a guide, I am including one that my student Rory made in October 2023.



Keep on playing at home!

Here are some activities that I encourage you to do over the next two weeks.

See if you have figured out the **pattern for counting diagonals**. Predict and then test if you're right for a 7-gon and 8-gon.

Make a **nice and neat Pascal's Triangle** of at least 10 rows. See if your family can make a few copies. Keep one copy to save and then….

See if you notice more patterns. Bring in your Triangle and a list of patterns to discuss.

On one copy **color all the even numbers one color and the odd numbers another**. See what you get. (This will be even more cool if you go with 12-15 or more rows. It's a lot of addition but fun practice!)

Do the same thing (the more rows the better) but color all 3s, 6s, 9s, etc. one color. Then pick a second color and color all 1s, 4s, 7s, etc. (again **skip counting by 3**) that color. Finally, pick a third color and color all 2s, 5s, 8s, etc. that third color. Are you convinced that this will cover every counting number 1, 2, 3, … that you might get in Pascal's Triangle? What do you get?

If you have more time and want to keep playing, do the same thing but **skip count by 4.** In other words, color with one color 1, 1+4=5, 5+4=9, 9+4=13, …; with another color 2, 2+4=6, ….. if you want you can continue. Wherever you stop, predict what will happen if you continue. I've included a 20-row Pascal's Triangle[2] on the last page for you to use if you wish.

I hope you had fun! See you next time!

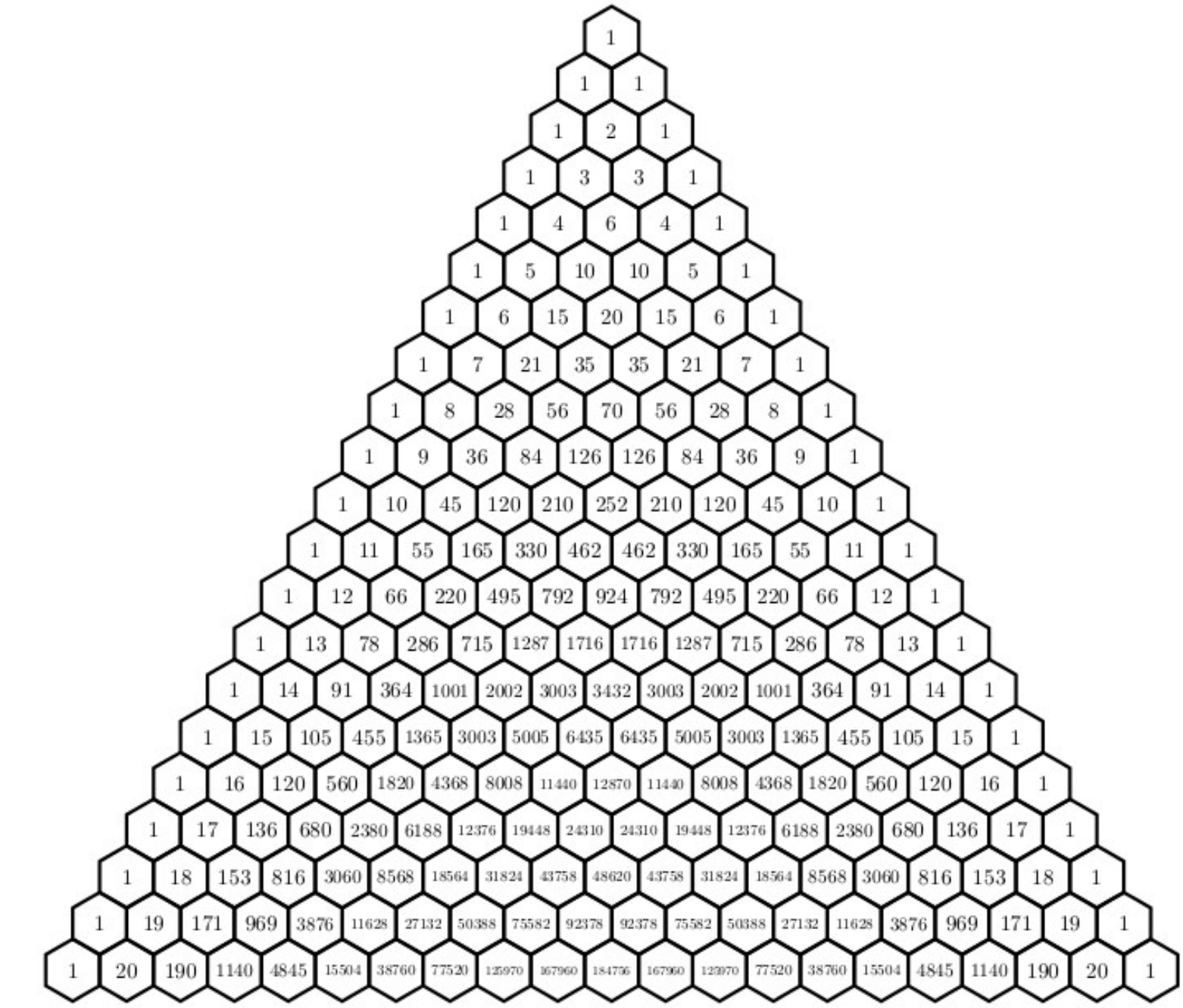
Have Fun!

Mr. Barman

P.S. Let your parents know that I host the country's largest vegetarian (all vegan) Thanksgiving! We used to sell out within minutes but now we open for lottery-style reservation requests. That opens as our class ends at 11:30a and runs through 11:30a on Monday – they can visit **trianglevegsociety.org**.

Also, please let them that Mathematics Institute of the Triangle is opening a new 3rd grade class; they can find out about our classes including 1st-3rd grade, middle school, prealgebra, algebra, geometry, and tutoring, at **triangelmathinstitute.com.**

[2] from tex.stackexchange.com/questions/198887/how-can-i-draw-pascals-triangle-with-some-its-properties, accessed Nov. 4, 2023


1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
1 13 78 286 715 1287 1716 1716 1287 715 286 78 13 1
1 14 91 364 1001 2002 3003 3432 3003 2002 1001 364 91 14 1
1 15 105 455 1365 3003 5005 6435 6435 5005 3003 1365 455 105 15 1
1 16 120 560 1820 4368 8008 11440 12870 11440 8008 4368 1820 560 120 16 1
1 17 136 680 2380 6188 12376 19448 24310 24310 19448 12376 6188 2380 680 136 17 1
1 18 153 816 3060 8568 18564 31824 43758 48620 43758 31824 18564 8568 3060 816 153 18 1
1 19 171 969 3876 11628 27132 50388 75582 92378 92378 75582 50388 27132 11628 3876 969 171 19 1
1 20 190 1140 4845 15504 38760 77520 125970 167960 184756 167960 125970 77520 38760 15504 4845 1140 190 20 1