Infinite Loops & Infinite Sequences

**loops, cylinders and Mobius strips ⇒ TOPOLOGY
infinite sequences ⇒ CALCULUS**

**Family Math Circle
2023**

Family Math Circle is an informal learning space where participants make advanced mathematics accessible to everyone in kind ways.

Roots and Wings

Make models to ground your math. Make connections to advance in higher math.

Make models of 3D shapes and connect to topology. Find and model examples of infinite sequences in our daily life, nature, and culture and connect to calculus.

Words With Math Friends

Tell friends and family all about your math creations. Use these terms +"math" to find images, videos, and articles on the web.

- ○ cylinder, Mobius strip, edge, side
- ○ loop, period
- ○ infinite, infinitesimal

- ○ to iterate, to limit, iteration, limit
- ○ periodic (repeating) sequence, not-periodic sequence

Choices, Choices

Mathematicians do many different things. What kind of math person will you be today?

Explore real sequences in nature, or fantastical and artistic sequences in culture. Make your own sequences or go on a scavenger hunt. Choose abstract math to imagine the impossible at the end of infinity, or choose applied math, science, and engineering to study realistically large and small numbers.

Toolbox

Physical (gray=optional):
- ∞ paper, colored pencils or markers
- ∞ scissors, glue or tape
- ∞ small toy to "travel the infinite roads"

Virtual:
- ∞ Bach on a Möbius Strip
  https://youtu.be/xUHQ2ybTejU
- ∞ *Walk Around in Circles* old song
  https://youtu.be/S0-coASIjkQ
- ∞ Encyclopedia of Integer Sequences
  https://oeis.org/

NaturalMath.com/circles/
Extra activities, videos, math connections, books, and other resources for math circle leaders.
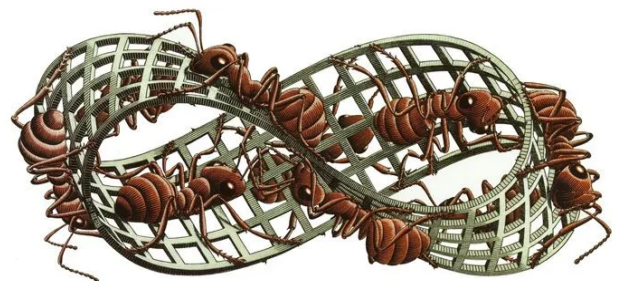
Mobius Strip II by M.C. Escher

Math Circle: Infinite Loops & Infinite Sequences

0. Warm-up: Walk Around in Circles

<u>Choose</u> a few toys you like. Place the toys around a big chair or table. Walk along this loop. Name each toy as you pass it. Repeat again and again—iterate!

Ask a math friend to call "Stop!" Can you remember what toy comes next?

Imagine going around forever. That's pure abstract math, impossible for real. If you choose to stay real, like an engineer, imagine going around for a very long time.

…cube, pyramid, bear, cube, pyramid, bear, cube, pyramid…

 You are building a model of a repeating infinite sequence.
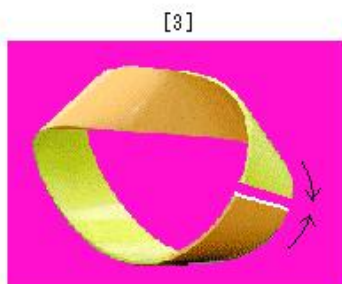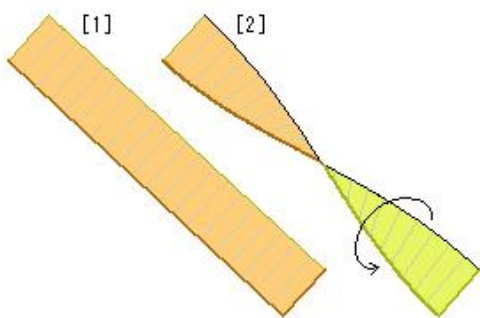
1. The Paper Loop

Cut out a strip of paper about an inch wide. Sketch the toys from your warm-up on your strip or write their names. Glue or tape the ends of your paper in a loop to make a <u>cylinder</u>.

Now you don't even have to walk when you iterate! Go around and around your loop with your finger. Do you get the same sequence if you go forward and backward?

Image: TheKatsMeow

2. A Loop with a Twist

Cut out a strip of paper again. This time, twist your strip, then glue it into a loop. Draw some pictures or shapes (or words or numbers) around your twisted loop. This loop is called <u>Mobius strip</u>.

Compare the Mobius strip to the paper cylinder you made before. Try these quick experiments with both shapes to grow your math eyes and notice their similarities and differences:

1. Place your pencil inside and roll the loop to draw a line, as if marking the middle of the road.
2. Place your pencil on the edge and color along the edge.
3. Poke a starter hole in the middle line to insert scissors, and cut along the middle line.
4. Take another strip of paper. Sketch a few toys or write a few words, numbers, or letters, as before. This time, write on each side. Glue or tape the ends to make a cylinder or a Mobius strip. Go around and around to make a repeating sequence. Does your sequence include everything you wrote?

Can you imagine a road twisted like a Mobius strip? How about a playground?
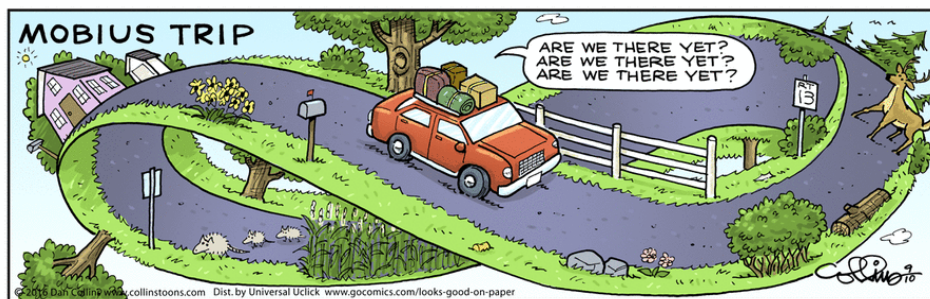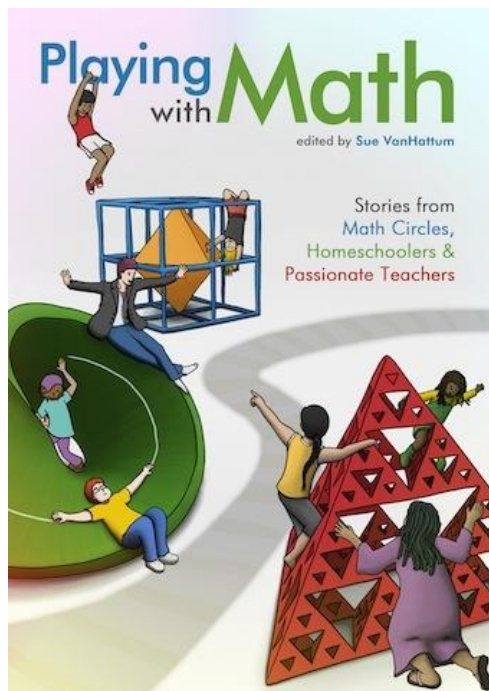
Images: Playing with Math, AlgeBruh123, Dan Collins

⚠ **SPOILER** Mobius Strip is famous. It appears in many fantasy and science fiction stories. It might surprise you.

3. Break the Loop

You might feel bored or even trapped if you imagine infinite iterations of the same few items. Luckily, some sequences go on and on but never repeat. You could count 1, 2, 3,... and on and on and on, and never repeat a number! Make these paper toys to explore sequences that don't repeat.
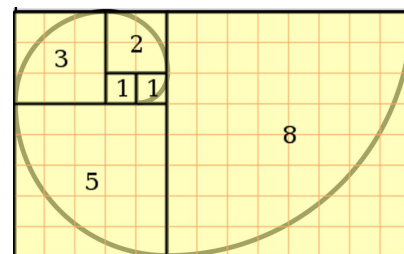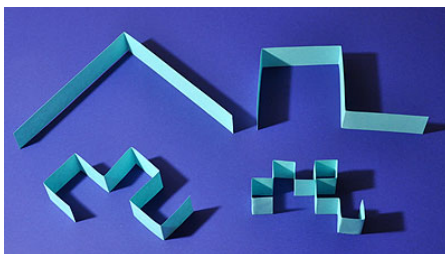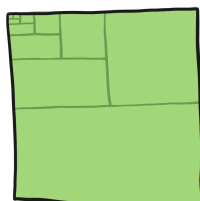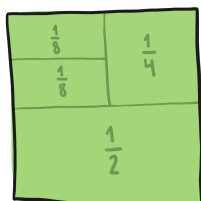
Images: Moebius Noodles; CutOutFoldPut.com

4. Your Own Infinite Sequences

Make a few <u>repeating sequences</u> of your own. We also call them <u>periodic</u>.
- ∞ Think of a few things that you do every day—or every week, or some other <u>period of time</u>. Draw or write them on a paper loop. Choose a cylinder or a Mobius Strip for your loop.
- ∞ Go on a scavenger hunt in nature. What repeats every day, year, or another period? Think of animal life, plants, planets…
- ∞ Do you know songs or stories with repeating parts?

Make a few <u>non-periodic sequences</u> of your own.
- ∞ Could you find or make a sequences where numbers grow bigger and bigger?
- ∞ Now find or make a sequence where numbers grow smaller and smaller!
- ∞ Find or make a sequence where numbers go up and down, up and down, but don't actually repeat.

MATH

IS WHAT YOU MAKE IT!

▶▶▶ **Name** _ _ _ _ _ _ _ _ _ _ _ _ _ ◀◀◀

~ Examples ~ Questions ~ Problems ~ Stories ~ Conjectures ~ Models ~ Art ~ Formulas ~ Graphs ~

▶▶▶ **Research Journal** ◀◀◀

Natural Math

By you and math friends