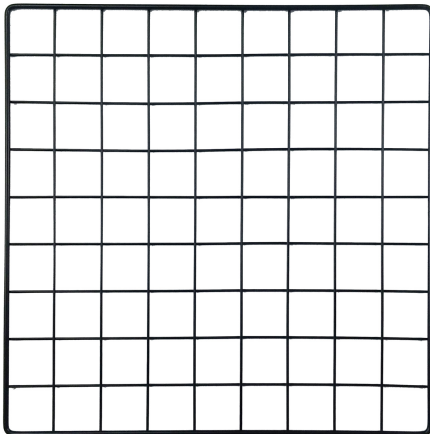# Euclid's Game

October 7, 2023

1 Euclid's Game - Catch 0 Version

Euclid's Game is a 2 player game, based on ideas from the Greek mathematician Euclid who lived over 2000 years ago . [1]

- Start with two numbers, e.g. 11 and 27.

- A move consists of taking away a positive multiple of the smaller number from larger number, so that the result is still non-negative. (If the two numbers are equal, just take one number away from the other number.)

- So from $(11, 27)$,

    - We can move to $(11, 16)$ by taking away 1×11 from 27.
    - Or we can move to $(11, 5)$ by taking away 2×11 from 27
    - Why can't we take away 3×11 from 27?
    - Are we allowed to away 0×11 from 27?

- The players take turns until one number becomes zero, so that the other player is out of moves and has lost.

- Here is a sample game: $(11, 27) \rightarrow (11, 5) \rightarrow (1, 5) \rightarrow (1, 3) \rightarrow (1, 0)$

1. For the game that starts with $(11, 27)$ and goes $(11, 27) \rightarrow (11, 5) \rightarrow (1, 5) \rightarrow (1, 3) \rightarrow (1, 0)$

    (a) How many moves were there in this game?
    (b) Did the first player win or the second player win?
    (c) Could the other player have won, from the same starting position of $(11, 27)$ if they made better moves?

2. Which player would you want to be, for the starting position $(11, 27)$, if you can make any moves you like that follow the rules?

3. How could you play this game with a token on a grid like this one?

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[1]This week's topic is from *https://theinnerframe.org/2021/01/03/euclids-game/*

2   Who Wins?

4. Play a few games from one of the following starting positions.

    (a) $(5, 1)$

    (b) $(3, 3)$

    (c) $(9, 2)$

    (d) $(5, 3)$

5. Play some more games with other starting positions.

3   Winning positions

6. Which starting positions are very boring because the first player wins no matter what?

7. Which starting positions are very boring because the second player wins no matter what?

8. Which starting positions could be won by either the first player or the second player depending on how the players move?

9. For which starting positions does Player 1 have a winning strategy – that is, Player 1 CAN win if they play the best moves, no matter what Player 2 does? We will call these "winning positions".

10. For which starting positions does Player 2 have a winning strategy – that is, Player 2 CAN win if they play the best moves, no matter what Player 2 does? We will call these "losing positions".

11. Color winning positions green and losing positions red on the grid.

12. What patterns do you notice as far as which squares are green and which are red?

13. What are the slopes of the boundary lines?