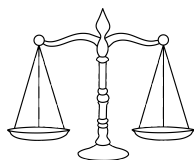


Weighing Puzzles ¹

All of these problems, unless otherwise stated, are about weighing coins using a balance scale with two pans.



A counterfeit coin is a fake one, and in these problems, the counterfeit coin is a different weight from the real ones.

1. Suppose you have 3 coins, and know that exactly one of them is counterfeit – lighter than the real ones. How can you find the counterfeit coin using a single weighing on a balance scale?
2. Suppose you have 3 coins, and know that exactly one of them is counterfeit – but you don't know whether it is lighter or heavier than the real ones! How can you find the counterfeit coin using just two weighings on a balance scale with two pans?
3. (a) You have 4 coins, and know that exactly one of them is counterfeit and lighter than the real ones. Find the counterfeit coin in two weighings on the balance scale.
(b) The same problem for 5 coins and two weighings.
(c) The same problem for 7 coins and two weighings
(d) The same problem for 9 coins and two weighings.
4. You have 4 coins. One of them is counterfeit – however, you don't know if it is lighter or heavier than the real ones. How can you find the counterfeit coin using just two weighings on a balance scale with two pans?
5. (a) There are 15 coins, and only one of them differs from the other (genuine) ones by weight. Can we determine whether this counterfeit coin is heavier or lighter than a genuine coin, using two weighings?
(b) Same question, for 16 coins.
(c) Same question, for 17 coins.

¹From *Mathematical Circle Diaries* by Ana Burago

6. There are 10 bags with coins. One of them contains only counterfeit coins, each of which is 1 gram lighter than a genuine coin. Using only one weighing on a balance with an arrow showing the difference between weights on the pans, find the bag that contains the counterfeit coins.
7. There are 6 coins, and two of them are counterfeit and are lighter than the genuine coins. Using three weighings, determine both counterfeit coins.
8. There are 10 bags with coins, and some of these bags contain only counterfeit coins. A counterfeit coin is 1 gram lighter than a genuine coin. One of the bags is known to be filled with the genuine coins. Using one weighing on a balance with one pan and with an arrow showing the weight on the pan, determine which bags contain counterfeit coins and which do not.
9. There are 5 coins, three of which are genuine. One is counterfeit and heavier than a genuine coin, and another one is counterfeit and lighter than a genuine coin. Using three weighings, find both counterfeit coins.
10. (a) There are 68 coins of different weights. Using 100 weighings, find the heaviest and the lightest of the coins.
(b) There are 64 stones of different weights. Using 68 weighings, find the heaviest and the second heaviest stones.
11. We have 6 weights: two green, two red, and two white. In each pair, one of the weights is heavier. All the heavy weights have the same weight, and all of the light weights have the same weight. Using two weighings, determine which weights are the heavy ones.
12. There are 6 coins, two of which are counterfeit – they are 0.1 gram heavier than the genuine coins. The pans of a balance are out of equilibrium only if the difference of weights is at least 0.2 grams. Find both counterfeit coins using four weighings.
13. (a) There are 16 coins. One of them is counterfeit: it differs in weight from a genuine coin, though we do not know whether it is heavier or lighter. Find the counterfeit coin using four weighings.
(b) There are 12 coins. One of them is counterfeit: it differs by weight from a genuine coin, though we do not know whether it is heavier or lighter. Find the counterfeit coin using three weighings.
14. Fourteen coins were presented in court as evidence. The judge knows that exactly 7 of these are counterfeit and weigh less than the genuine coins. A lawyer claims to know which coins are counterfeit and which are genuine, and she is required to prove it. How can she accomplish this using only three weighings?