Symmetry Groups

1. Given the dots below, how many ways can we swap the dots?

   (a)

   (b)

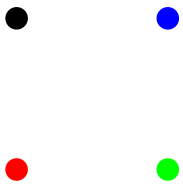
   (c)

   (d) How many ways can we swap n dots?

2. When we swap dots around without any additional constraint, the collection of all swaps is called the "Symmetric Group of n Objects." How can we list out all the different types of swaps?

3. Suppose that we have 4 dots below:

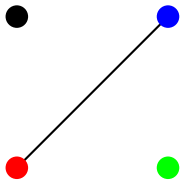
   (a) List some symmetries of this object. Describe them in detail, including the property that is changing and the property that is staying the same.

   (b) Suppose that we are allowed to swap the dots around in any way we want. Describe the symmetry here.

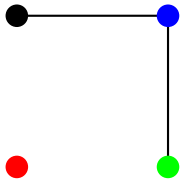
   (c) How many different ways can we swap the dots?

4. Do parts (b) and (c) the above, but instead with these objects where when you switch the dots, the lines must stay intact. Do you notice any patterns, relationships with these symmetries?

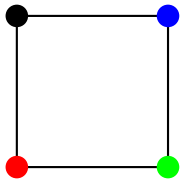
(a)



(b)

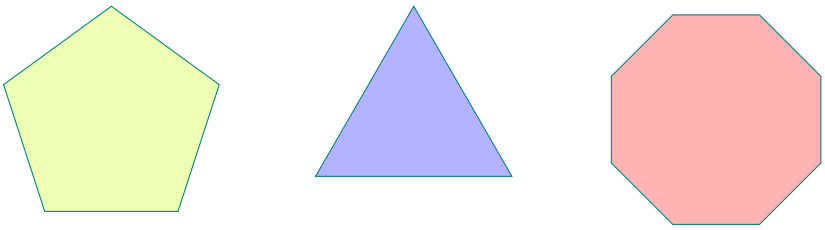


(c)



5. For part (c) above, how can we describe all of the possible swaps?

6. This last question can be though of as finding the geometric symmetries of a square. Use the same strategy to find the collection of symmetries for the following polygons.



These collections are called the "Dihedral Groups."