1 Clocks and Calendars

1. (a) Suppose it's 4PM and someone wants to meet you in 5 hours. When is your meeting time?
   (b) Suppose it's 10AM and someone wants to meet you in 5 hours. When is your meeting?
   (c) Suppose it's 9PM and you have a meeting in 12 hours. When is your meeting?
   (d) Suppose it's 1PM, and you have a meeting in 19 hours. What time is your meeting?
2. (a) 20 minutes after 5:47 is 6:_____
   (b) 37 minutes after 3:52 is 4:_____
3. Today, January 22, 2022 is a Saturday. There are 31 days in January and 28 days in February.
   (a) What day of the week will January 31 be?
   (b) What day of the week was January 6?
   (c) What day of the week will March 31 be?

Doomsday Rule

It turns out that the following dates are always on the same day of the year. We will call that day "Doomsday".

- The last day of February
- 4/4
- 6/6
- 8/8
- 10/10
- 12/12
- 5/9
- 9/5
- 7/11
- 11/7

This year (2022) Doomsday is on a Monday. So all of those dates are Mondays in 2022.

4. Use the fact that Doomsday is a Monday to determine what day the following dates fall on in 2022.
   (a) March 7
   (b) August 10
   (c) September 20
   (d) Halloween
   (e) Christmas
   (f) Thanksgiving
   (g) Your Birthday

2 Modulo

For two numbers A and B, we say that $A \equiv B \pmod{12}$ if A and B have the same remainder when divided by 12.

For example, $15 \equiv 3 \pmod{12}$.

Fill in the blank with a small number: $\_\_\_\_\_ \equiv 21 \pmod{12}$?

For two numbers A and B, we say that $A \equiv B \pmod{5}$ if A and B have the same remainder when divided by 5.

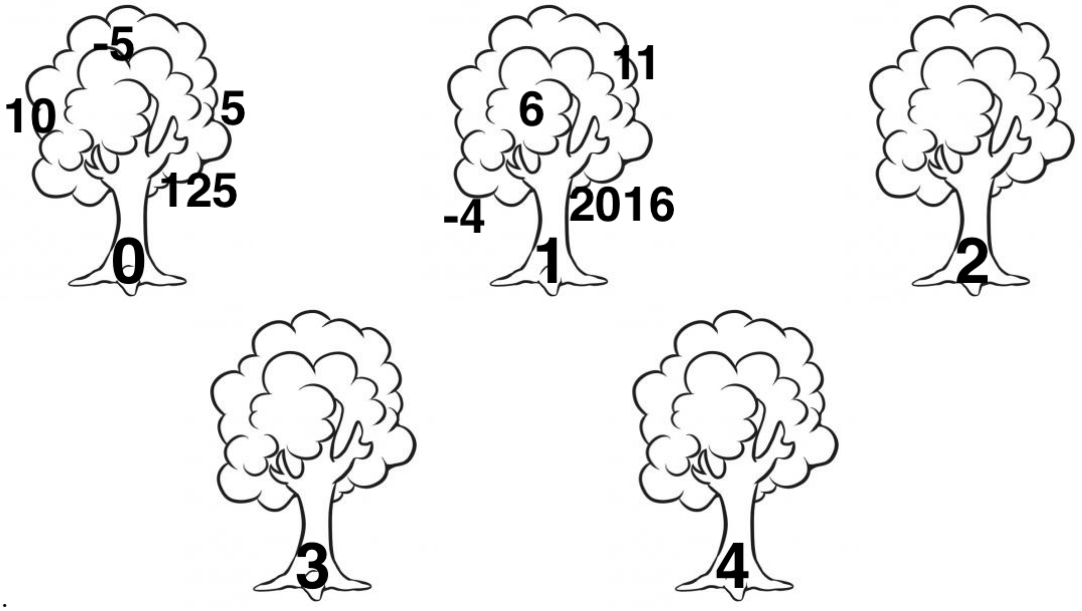
- $8 \equiv 23 \pmod{5}$. Why?
- $8 \not\equiv 14 \pmod{5}$. Why not?

5. (a) Is $13 \equiv 6 \pmod{5}$?
   (b) Is $85 \equiv 0 \pmod{5}$?
   (c) Is $17 \equiv 3 \pmod{7}$?
   (d) Is $5 \equiv 2 \pmod{4}$?
   (e) Is $4 \equiv -1 \pmod{5}$?

3 Mod n Trees

6. Fill in the blanks with the smallest positive numbers possible.
   (a) $76 \equiv$ _____ (mod 12)
   (b) $52 \equiv$ _____ (mod 12)
   (c) $76 \equiv$ _____ (mod 60)
   (d) $15 \equiv$ _____ (mod 7)
   (e) $15 \equiv$ _____ (mod 3)
   (f) $15 \equiv$ _____ (mod 11)

7. Here is a drawing of the world (mod 5). On the tree with a 0 on the trunk, we put all the numbers that are congruent to 0 (mod 5). On the tree that with a 1 on the trunk, we put all the numbers that are congruent to 1 (mod 5). Write at least four numbers on each of the other trees.



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8. For the (mod 5) trees above, is it possible to have the same number on two different trees?

9. Draw trees for (mod 2). How many trees do you need? Draw at least four numbers on each tree. What word could you use to describe the numbers on the 0 tree (mod 2)? On the 1 tree (mod 2)?

10. Draw trees for (mod 3).

11. Draw trees for (mod 4). Is there a relationship between the trees (mod 4) and the trees (mod 2)?

4 Adding and Multiplying (mod n)

12. Match the arithmetic problems on the left and the right that give the same answers.

$10 + 15 \pmod 7$	$5 + 1 \pmod 7$
$12 + 22 \pmod 7$	$10 \pmod 7 + 15 \pmod 7$
$15 \times 22 \pmod 7$	$15 \pmod 7 \times 22 \pmod 7$
$14 \times 144 \pmod 7$	$1 \pmod 7 \times 1 \pmod 7$
	$0 \times 4 \pmod 7$

13. Compute these sums. Hint: you don't need to do a lot of arithmetic.
    (a) $423 + 577 \pmod{10}$
    (b) $56 + 89 \pmod{10}$
    (c) $892 + 9823 \pmod 5$
    (d) $901 + 723 \pmod 3$

14. Compute these products. Hint: be lazy.
    (a) $4893 \times 49024 \pmod{10}$
    (b) $3982734 \times 2398739 \pmod{10}$
    (c) $78 \times 23 \pmod 5$
    (d) $3874 \times 3284 \pmod 3$

5 Last digits

15. (a) What is the last digit of $14,306 + 908,797$? Can you find the answer quickly, without doing the whole addition problem?

    (b) What is the last digit of 5589×4523?

    (c) What is the last digit of $413 \times 5967 \times 4534$?

16. What is the last digit of 3^{2022}? Remember, 3^{2022} means we multiply 3 by itself 2017 times. Hint: try to find a pattern by finding the last digit of 3^1, 3^2, 3^3, etc.

17. What is the last digit of 9^{99}?

18. What is the last digit of 2^{100}?