Rational Tangles

1 Warm-Up

1. Negative Reciprocals

   Example 1: 4/3 is the negative reciprocal of -3/4

   Example 2: -2/3 is the negative reciprocal of 3/2.

   Find the negative reciprocal of (a) 5/11   (b) -7/3

2. Fraction practice: Remember how to add a fraction and a whole number:

   Example: -3/5 + 1 = -3/5 + 5/5 = 2/5

   Now you try it:

   (a) -9/5 + 1 =.   (b) -5/2 + 1 =   (c) -7/4 + 2 =

   (d) -13/3 + 5 =

2 Rational Tangles

We will tangle up two ropes, held at the ends by four people, by repeatedly using only two moves: Twist (T) and Rotate (R).

- For Twist, the two people on the right (from the point of view of the people in the class) change places, with the person initially in the rear lifting his or her rope and the person in front stepping under it.
- For Rotate, the four people rotate 90° in the clockwise direction (as viewed from above).

We will also use a third move: Display (D).

3. We would like to associate a number with each tangle that we can get by performing these two moves.

   - What would be a reasonable number to assosciate with the starting position, with no crosses?

   - What number should be associated to the tangle we get by then doing 1 Twist? 2 Twists?

4. How does a Twist (T) change a tangle's number?

5. How does Rotate (R) change a tangle's number?

6. What numbers do you get from these moves?

    (a) TTRTTRTTRT
    (b) TTTRT
    (c) TTRTT
    (d) TTTTR

7. Suppose you have the tangle 2. Can you use T and R to untangle it to 0?

8. Try getting from these numbers back to 0, using only T and R,

    (a) $\dfrac{1}{4}$

    (b) $\dfrac{2}{3}$

    (c) $\dfrac{5}{3}$

    (d) $-\dfrac{1}{4}$

9. Is it possible to use T and R to get from ANY number back to 0?

10. Is it possible to use T and R to get from 0 to any other number?

11. Can you find a sequence of T's and R's that is equivalent to an UnTwist?

3   Euclidean Algorithm

The Euclidean Algorithm is a method for finding the greatest common divisor of two numbers.

12. What is the greatest common divisor of 105 and 75? How do you find it?

13. Use the Euclidean algorithm to find the gcd of

   (a) 46 and 115

   (b) 323 and 117

   (c) 112 and 51

   (d) 22 and 53

   (e) 17 and 7

14. What rational tangles do you go through in converting the rational tangle

   (a) $\frac{22}{53}$ to 0?

   (b) $\frac{7}{17}$ to 0?

15. Convert the rational tangle $\frac{51}{112}$ to 0. Then use the Euclidean algorithm to find the gcd of 112 and 51, but each time, overshoot.

4   Rational Tangle Construction

16. Find sequences of T's and R's that can produce each of the fractions in this table with a minimum number of steps. Can you find any patterns? When filling out the table, use the number at the top of the column as the numerator and the number at the start of the row as the denominator, so that $5/7$ is in the 5th column and 7th row.

Positive Fractions

	1	2	3	4	5	6	7
1							
2							
3							
4							
5							
6							
7							

Negative Fractions

	-1	-2	-3	-4	-5	-6	-7
1							
2							
3							
4							
5							
6							
7							