Adding, Adding, and more Adding, Dilip Barman

This is material that I developed in September 2021 for the Chapel Hill Math Circle's Intermediate Group (recommended for grades 6-8).

1. What is 1+2+3+4+ … + 10? How about 1+2+3+ … + 100?

2. What if we want to add to a number in general? Pick a positive integer n. What is 1+2+3+ … + n?

3. Try these sums.

    a. -25 + -24 + -23 + … + -1 + 0

    b. -53 + -52 + -51 + … + -1 + 0 + 1 + 2 + … 80

    c. 3 + 6 + 9 + 12 + … + 3000

4. What is the sum of the first n (positive integer) odd numbers?

5. What is the sum of the first n (positive integer) even numbers?

6. Let's review exponents.

   a. An exponent just tells us how many times to multiply a number called the base. For example, $3^4 = 3 \times 3 \times 3 \times 3 = 81$. Write down the powers of 2 and the powers of 3 from 1 to 5. Based on this, what would 2^0, 2^{-1}, 2^{-2}, and 3^0, 3^{-1}, 3^{-2} be?

   b. How could you sum successive powers of 2? Pick a positive integer n; what is $2^0 + 2^1 + 2^2 + \ldots + 2^n$?

   c. What about successive powers of 3?

   d. Can you generalize to summing the powers of a positive integer b, $b^0 + b^1 + b^2 + \ldots + b^n$?

   Now it's your turn. Come up with an interesting sequence of numbers. Can you find out its sum? How about ½ + ¼ + 1/8 + 1/16 + … going on forever? How about other infinite sequences? Let's discuss and work on some of these together, time permitting.

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