Knots
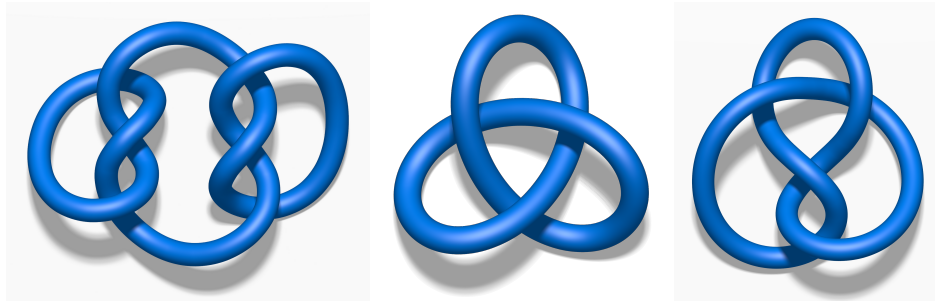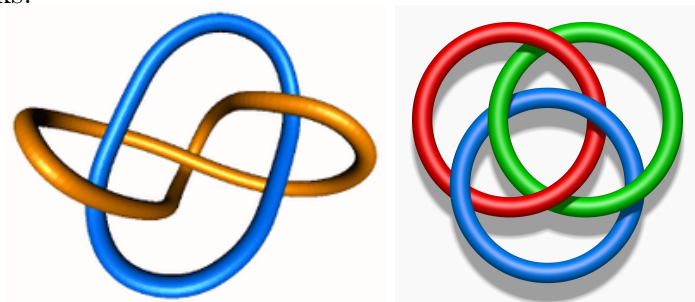**From Maia Averett and _A Decade of the Berkeley Math Circle, Volume 2_**

1 Warm-Up

Loosely speaking, a knot is a piece of string that has been tangled up and then had the ends fused together.

When the knot is made from two or more loops of string, it is called a link. Here are some examples of links.
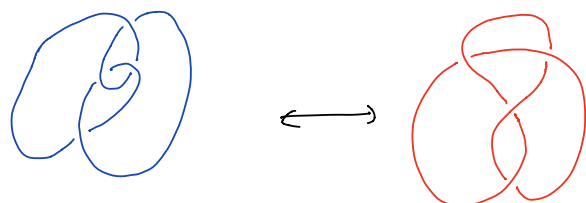
Warm Up Activity:

- Use your pipe cleaners to create some of the knots above.

- Try making some knots of your own (you can fuse pipe cleaners together to make the knots larger.

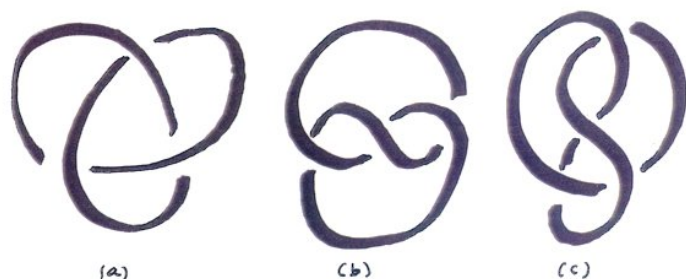- If you have any questions/observations about knots (or anything else), write them down!
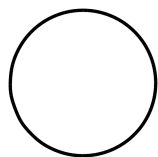
1. In what sense are the following knots equivalent?
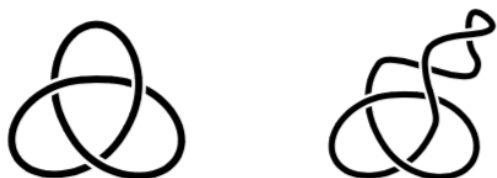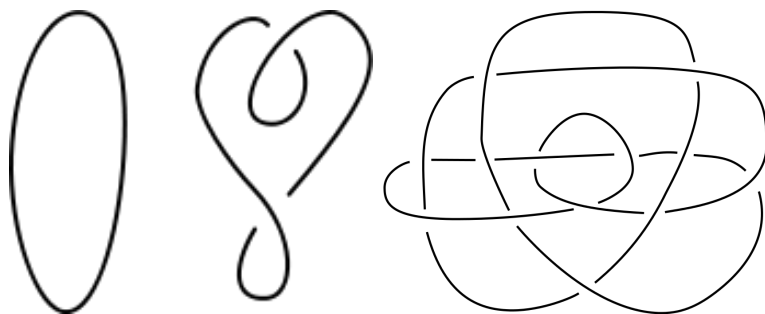
2. Are these knots equivalent?

3. Which of these knots, if any, are equivalent?

   (a)   (b)   (c)

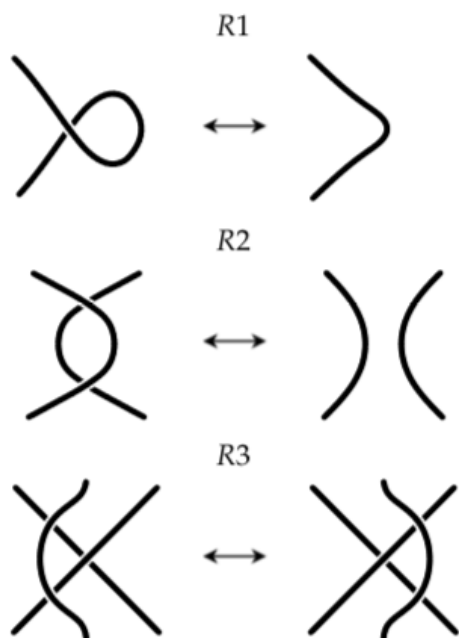   **Definition:** An *unknot* is a knot equivalent to this one:
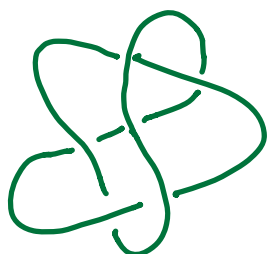
4. Which of these are unknots?

2   Reidemeister Moves

Definition: A Reidmeister move is one of the following three moves:

Each move has several variations. For example, in a type R3 move, the strand might be entirely under instead of over the crossing.

5. Use a sequence of Reidmeister moves to get from this knot diagram to the standard diagram for the unknot.

6. Suppose we can get from one knot diagram to another by a sequence of Reidmeister moves. Are the two knots represented by the two knot diagrams necessarily equivalent?

7. Suppose we have two knots that are equivalent, but are represented by two different knot diagrams. Can we get from one knot diagram to the other by a sequence of Reidmeister moves?