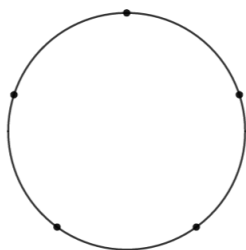# Relatively Prime Stars

1. Draw 5 points around a circle, evenly spaced. Start at a point, and going around the circle, connect it to a point 2 points over. Repeat until you're back at the starting point. You should get a 5-pointed star.

2. Now do the same thing, starting with 8 points around a circle, each time connecting to a point 2 points over. If you get back to your starting point before using up all the points, find a lonely point and start again with a different color.
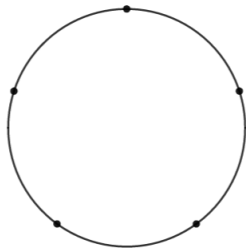
3. Experiment with different numbers of points around the circle "P" and different numbers of points that you go over "Q". Here is the beginning of a star where P = 8 and Q = 3.
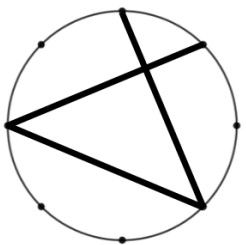
   Any time you get back to your starting point, if there are any dots left, start a new line with a different color and keep track of the number of colors.

4. Record your observations here.

P	Q	Number of colors
5	1	1
8	2	2

   (a) For what values of P and Q do you end up connecting all the points?

   (b) For what values of P and Q do you end up getting back to your starting point before connecting all the points?

5. Without drawing the stars, can you predict how many colors will be needed and how many points there will be on each star?

   (a) if $P = 7$ and $Q = 4$?

   (b) if $P = 15$ and $Q = 3$?

   (c) if $P = 15$ and $Q = 9$?

   (d) if $P = 14$ and $Q = 4$?

   (e) if $P = 35$ and $Q = 10$?

6. What values of P and Q make the best looking stars?

Pouring Water

1. Given a 5-liter unmarked container, a 3-liter unmarked container, and an unlimited supply of water, can you obtain an accurate measure of 4 liters of water?

   You can keep track of your pourings here.

5-liter	3-liter
0	0
5	0

5-liter	3-liter
0	0
0	3

   Could you also obtain an accurate measure of 1 L of water? 2 L? 3 L? 6 L?

2. Given a 4-liter unmarked container, a 7-liter unmarked container, and an unlimited supply of water, can you obtain an accurate measure of 5 liters of water? If so, what is the minimum number of pourings necessary?

4-liter	7-liter
0	0
4	0

4-liter	7-liter
0	0
0	7

   Could you also obtain an accurate measure of 1 L of water? 2 L? 3 L? 6 L?

3. Under the same conditions, but with a 3-liter container and a 6-liter container, can you obtain a measure of 5 liters? If so, what is the minimum number of pourings necessary?

3-liter	6-liter	3-liter	6-liter
0	0	0	0
3	0	0	6

4. Given a 5-liter unmarked container, a 9-liter unmarked container, and an unlimited supply of water, can you obtain an accurate measure of 6 liters of water? If so, what is the minimum number of pourings necessary?

5-liter	9-liter	5-liter	9-liter
0	0	0	0
5	0	0	9

5. On the pool table shown below, a pool ball must be shot initially from the point $(0,0)$ and must roll along the side to either $(0,3)$ or $(5,0)$. Each time the ball strikes another side, it will bounce off at an angle of $60°$ as indicated by the dotted lines. First, start at $(0,0)$, and shoot the ball towards $(5,0)$, and record the coordinates each time the ball strikes a side. Then start at $(0,0)$, shoot the ball towards $(0,3)$, and record the coordinates again. Can you relate your results to the previous problem?