# Rational Tangles

## 1 Warm-Up

1. Reduce the fraction $\frac{217}{713}$ Hint: try flipping fractions.

## 2 Rational Tangles

We will tangle up two ropes, held at the ends by four people, by repeatedly using two moves: Twist (T) and Rotate (R).

- For Twist, the two people on the right (from the point of view of the people in the class) change places, with the person initially in the rear lifting his or her rope and the person in front stepping under it.
- For Rotate, the four people rotate 90° in the clockwise direction (as viewed from above).

We will also use a third move: Display (D).

2. We would like to associate a number with each tangle that we can get by performing these two moves.
   - What would be a reasonable number to associate with the starting position, with no crosses?
   - What number should be associated to the tangle we get by then doing 1 Twist? 2 Twists?
3. How does a Twist (T) change a tangle's number?
4. What is the tangle associated with the number 3? The number $-3$?
5. How does Rotate (R) change a tangle's number?
6. Suppose you have the tangle 2. Can you use $T$ and $R$ to untangle it to 0?
7. Try getting from these numbers back to 0:
   (a) $\frac{1}{4}$
   (b) 3
   (c) $\frac{5}{3}$
8. Is it possible to use $T$ and $R$ to get from ANY number back to 0?
9. Reduce the fraction $\frac{115}{506}$ to lowest terms by finding the greatest common factor using the Euclidean algorithm. What does this have to do with rational tangles?

10. Reduce the fractions by finding the greatest common denominator using the Euclidean algorithm and by using rational tangle moves.
    (a) $\frac{187}{323}$
    (b) $\frac{217}{713}$
11. Is it possible to use $T$ and $R$ to get from 0 to any other number?
12. Can you find a sequence of $T$'s and $R$'s that is equivalent to an UnTwist?
13. Make a rational tangle, and then remove one rope while having the dancers on the other rope hold on tight. Is it possible for the result to have a knot in it?

14. Find sequences of $T$'s and $R$'s that can produce each of the fractions in this table with a minimum number of steps. Can you find any patterns? When filling out the table, use the number at the top of the column as the numerator and the number at the start of the row as the denominator, so that 5/7 is in the 5th column and 7th row.

Positive Fractions

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 1 | | | | | | | |
| 2 | | | | | | | |
| 3 | | | | | | | |
| 4 | | | | | | | |
| 5 | | | | | | | |
| 6 | | | | | | | |
| 7 | | | | | | | |

Negative Fractions

| | -1 | -2 | -3 | -4 | -5 | -6 | -7 |
|---|---|---|---|---|---|---|---|
| 1 | | | | | | | |
| 2 | | | | | | | |
| 3 | | | | | | | |
| 4 | | | | | | | |
| 5 | | | | | | | |
| 6 | | | | | | | |
| 7 | | | | | | | |