Two is the Best Number (Part 3)

1 Warm-Up

1. In this problem there is a long straight road.

   (a) A cricket jumps along the road. Every jump takes him 1 meter to the right or to the left. Could it be that in 21 jumps he will end up exactly where he started? Explain.

   (b) A brown cricket and a green cricket are sitting on the road 9 meters apart. They start jumping at exactly the same moment. On every jump, each of them leaps 1 meter to the right or to the left. Whenever one jumps, the other jumps too, at the same time. Can they ever land on the same spot simultaneously? Explain.

   (c) A brown cricket and a green cricket are sitting on the road 15 meters apart. They start jumping at exactly the same moment. On every jump, each of them leaps 3 meters to the right or to the left. Whenever one jumps, the other jumps too, at the same time. Can they ever land on the same spot simultaneously? Explain.

2 Nice to Meet You

2. Divide up into groups of three, four, or five, and shake hands an odd number of times with people in your group. Keep track of how many times you shake hands. You can shake hands with the same person more than once, but you cannot shake hands with yourself. Can you do it?

   Note: for pandemic times, instead of actually shaking hands with someone, you can send them a private chat "handshake".

   - If you message another student "handshake" and the student doesn't respond, then that counts as one handshake for BOTH students.

   - If another student messages you "handshake" and you don't respond, that counts as one handshake for both of you.

   - If you message another student "handshake" AND the other student messages back "handshake" (or vice versa) then that counts as TWO handshakes for EACH of you.

3. Suppose you shake hands with each other person in your group once and only once. How many total handshakes will there be ... if your group contains 4 people? 7 people? n people?

3 Extra Problem: Evil Wizard

4. An evil wizard has imprisoned 64 math circle participants. The wizard announces, "Tomorrow I will have you stand in a line, and I will put a hat on each of your heads. The hat will be colored either white or black. You will be able to see the hats of everyone in front of you, but you will not be able to see your hat or the hats of the people behind you. I will begin by asking the person at the back of the line to guess his or her hat color. If the guess is correct, that person will get a cookie. If the guess is wrong, that person will be killed in a painful way. Then I will ask the next person in line, and so on. You are only allowed to say the single word 'black' or 'white' when it is your turn to speak, and otherwise you are not allowed to communicate with each other while you are standing in line. Although you will not be able to see the people behind you, you will know (by hearing) if they have answered correctly or not."

   The prisoners are allowed to chat for a few minutes before their ordeal begins. What is the largest number of prisoners that can be guaranteed to survive?

4   Extra Problem: Cat and Mouse

5. A very polite cat chases an equally polite mouse. They take turns moving on the grid depicted below.
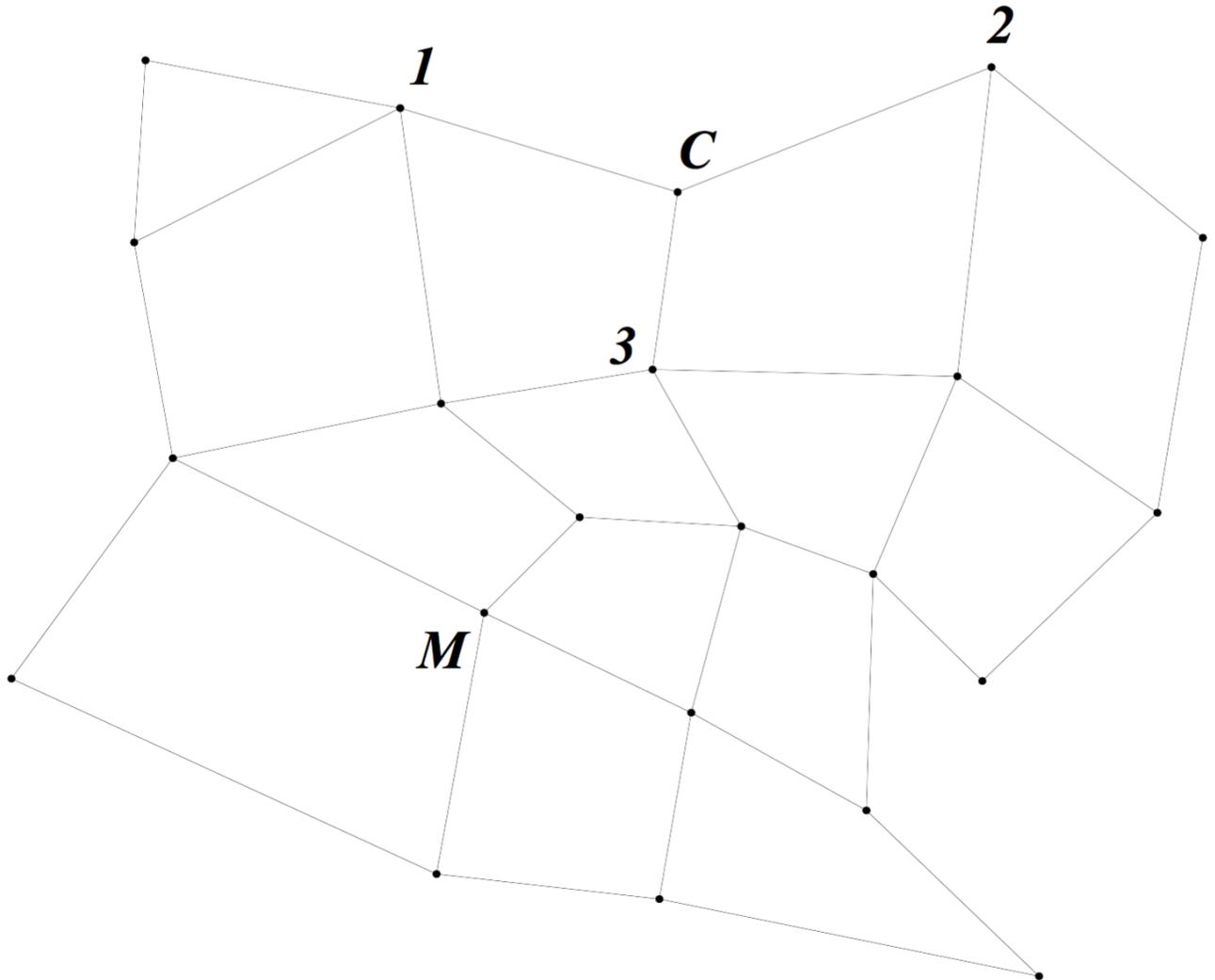
Initially, the cat is at the point labeled C; the mouse is at M. The cat goes first, and can move to any neighboring point connected to it by a single edge. Thus the cat can go to points 1, 2, or 3, but no others, on its first turn. The cat wins if it can reach the mouse in 15 or fewer moves. Can the cat win?