Two is the Best Number (Part 2)

1 Warm-Up

1. A chess rook stands at the bottom left corner of a 5 x 5 chess board – the one marked with a star in the figure. Being lazy, this rook does not like to move far – in a single step it moves only one square up, down, right, or left. For exercise, the rook is supposed to tour the entire chessboard, visiting each square just once.

   (a) Can the rook end its walk on the square marked with a flower?

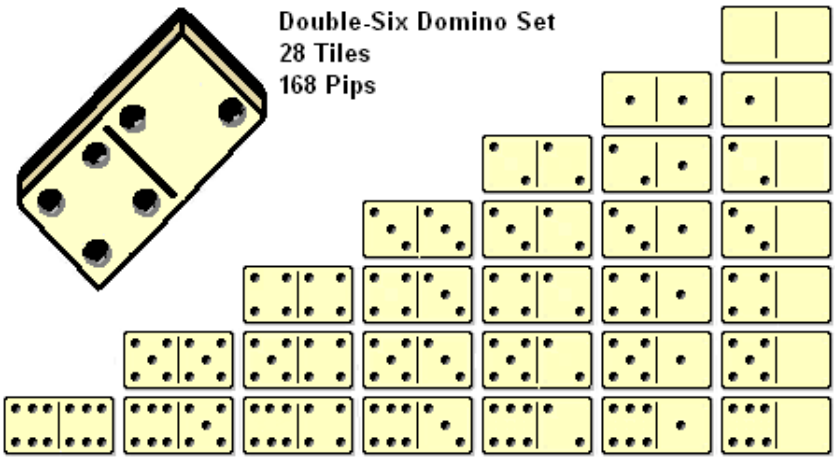
   (b) Can the rook end its walk on the square marked with a smiley face?

   (c) Can the rook end its walk back next to the square with the star?

2 Dominos

2. Kolya placed a full set of dominos (all 28 pieces) into a line according to the rule of dominos (see below). The left half of the leftmost domino has 6 spots on it. How many spots are there on the right half of the rightmost domino?

   Rule of Dominos: When two dominos are placed next to each other, end to end, the halves that are next to each other must have the same number of spots.



3 Nice to Meet You

3. Divide up into groups of three, four, or five, and shake hands an odd number of times with people in your group. Keep track of how many times you shake hands. You can shake hands with the same person more than once, but you cannot shake hands with yourself. Can you do it?

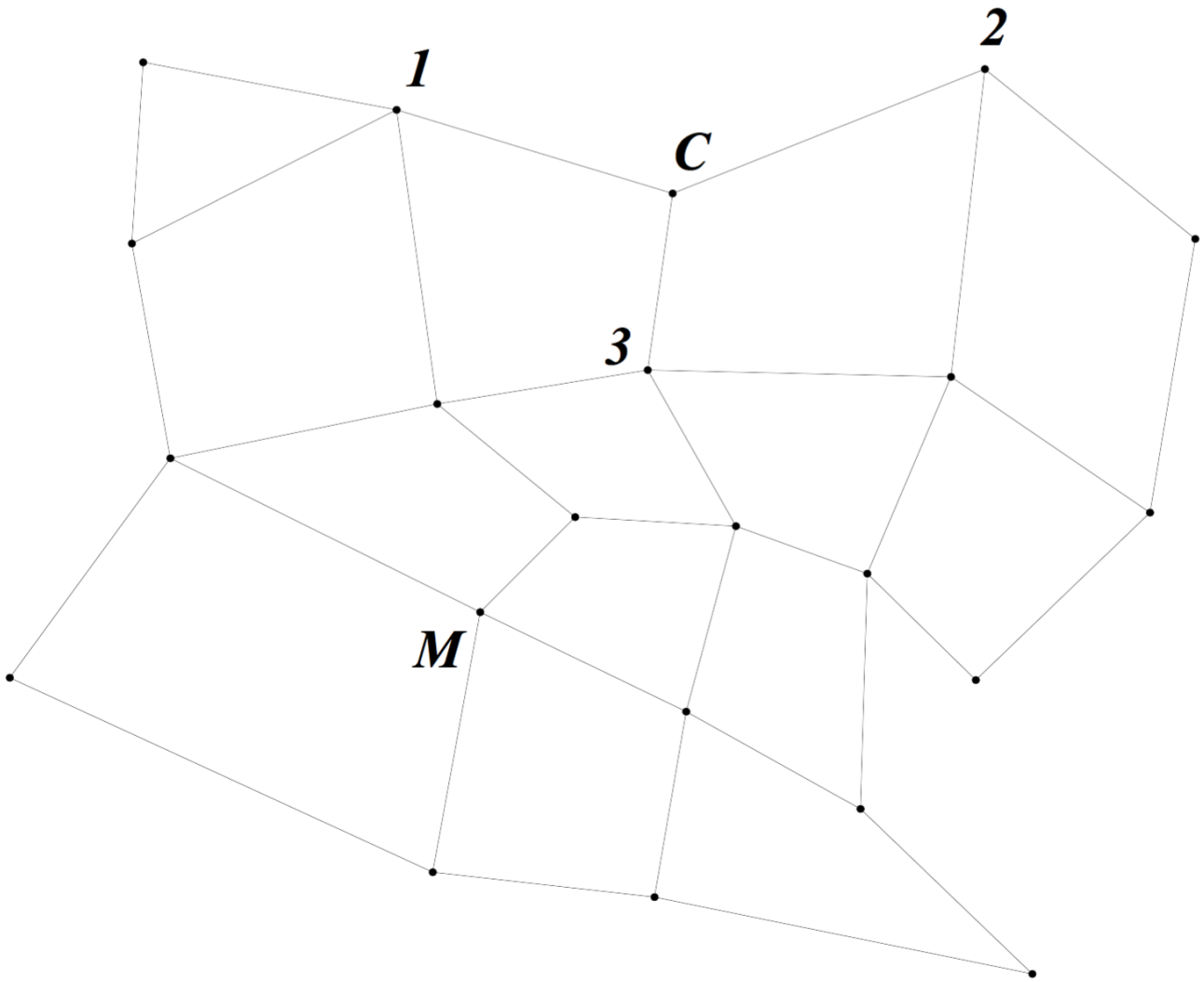
   Note: for pandemic times, instead of actually shaking hands with someone, you can send them a private chat "handshake" and that will count as a handshake for both of you. If you message the "handshake" AND they message you back "handshake" then that counts as two handshakes for both of you.

4. Suppose you shake hands with each other person in your group once and only once. How many total handshakes will there be ... if your group contains 4 people? 7 people? n people?

Many of this week's problems are from *Mathematical Circle Diaries, Year 1* by Anna Burago.

4 Problems for Next Time

5. A very polite cat chases an equally polite mouse. They take turns moving on the grid depicted below.



Initially, the cat is at the point labeled C; the mouse is at M. The cat goes first, and can move to any neighboring point connected to it by a single edge. Thus the cat can go to points 1, 2, or 3, but no others, on its first turn. The cat wins if it can reach the mouse in 15 or fewer moves. Can the cat win?

6. An evil wizard has imprisoned 64 math circle participants. The wizard announces, ÒTomorrow I will have you stand in a line, and I will put a hat on each of your heads. The hat will be colored either white or black. You will be able to see the hats of everyone in front of you, but you will not be able to see your hat or the hats of the people behind you. I will begin by asking the person at the back

of the line to guess his or her hat color. If the guess is correct, that person will get a cookie. If the guess is wrong, that person will be killed in a painful way. Then I will ask the next person in line, and so on. You are only allowed to say the single word ÔblackÕ or ÔwhiteÕ when it is your turn to speak, and otherwise you are not allowed to communicate with each other while you are standing in line. Although you will not be able to see the people behind you, you will know (by hearing) if they have answered correctly or not.Ó

The prisoners are allowed to chat for a few minutes before their ordeal begins. What is the largest number of prisoners that can be guaranteed to survive?

7. The integers from 1 to 18 are written on the board in a row. Can you insert plus and minus signs between them in such a way as to get an expression that is equal to zero?