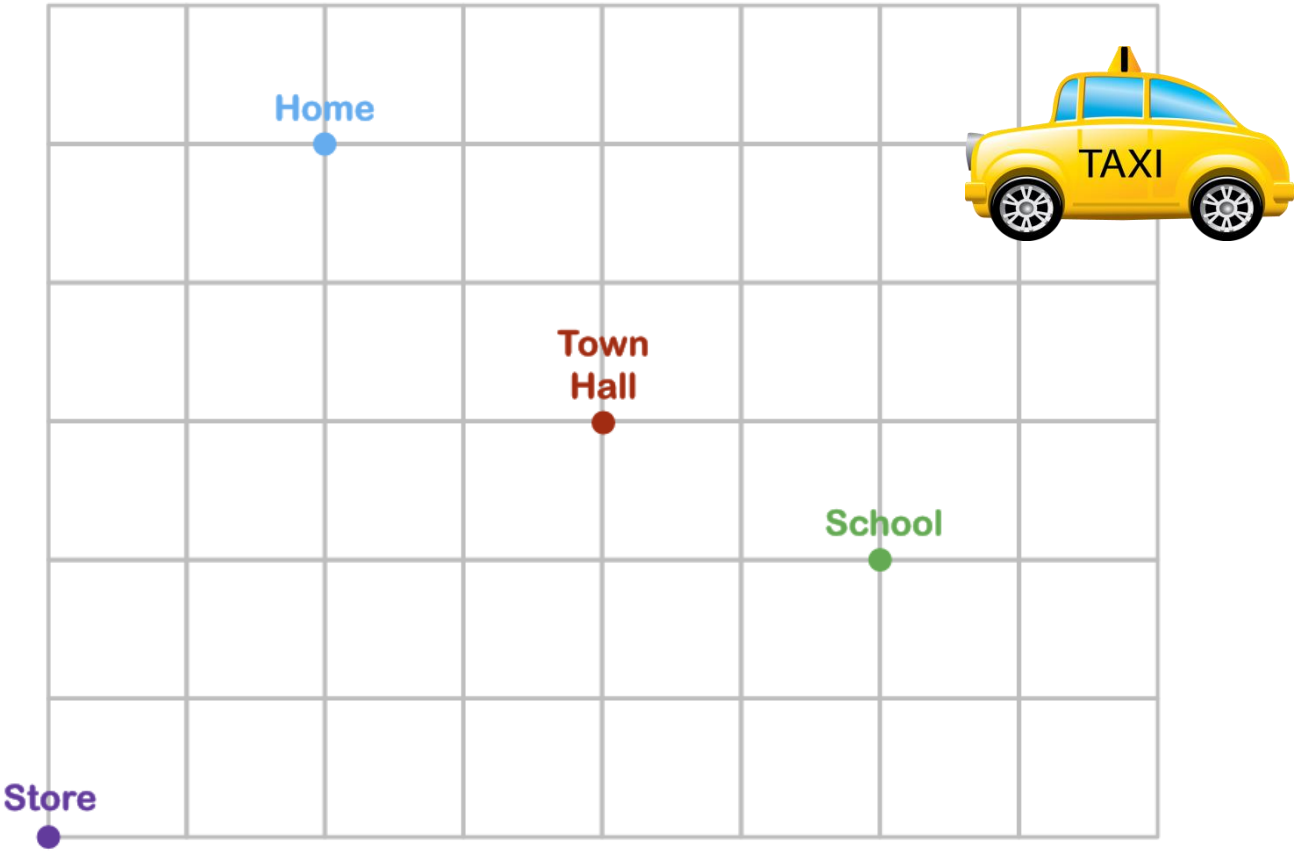
Taxicab Geometry, Dilip Barman

Chapel Hill Math Circle, Intermediate Group, February 20, 2021

This is material that I developed in May 2020 based on an interesting Math Teachers' Circle Workshop that I had attended. I made some updates for our February 2021 Math Circle for the Intermediate Group (recommended for grades 6-8). I hope that you will enjoy it!

Imagine that you have a relative who visits you from England to attend UNC. They arrive a few days early to visit with you and inquire how to drive to campus from your home, say in Raleigh. Would the fact that they drive on the other side of the road than we do affect your directions?

Have you ever visited a big city where the streets form a rectilinear grid? Let's consider such a city.



Assume that each “city block” is 1 mile by 1 mile for simplicity of calculations. What does *distance* mean? How far apart are the town hall & store “as the crow flies”? How about by car?

1. If you are driving, what is the distance between the town hall and the school? How many ways (non-wasteful ways) can you get to the school from town hall?
2. How about from home to town hall?
3. Let’s look at a bigger problem; how many (non-wasteful) ways are there to drive from town hall to the store? Is there an organized way that you can calculate this?
4. How could you model a hybrid such as “bicycle geometry”? As a bicyclist, you follow normal rules of the road. But at certain situations, such as a prolonged red light or bicycle paths cutting across blocks, perhaps your bicycle could be walked and you could be a pedestrian?
5. What does a “circle” look like in “taxicab geometry”? Draw such a circle of radius 3 miles centered on town hall.
6. (Extra) If we define “Ti” to be circumference divided by diameter in “taxicab geometry”, what is its value here? Is it always the same?

When we look at geometry, we usually look at Euclidean geometry that has five basic assumptions (postulates). This has been a little peek into a non-Euclidean geometry where some of the assumptions no longer hold; this example is “taxicab geometry.”