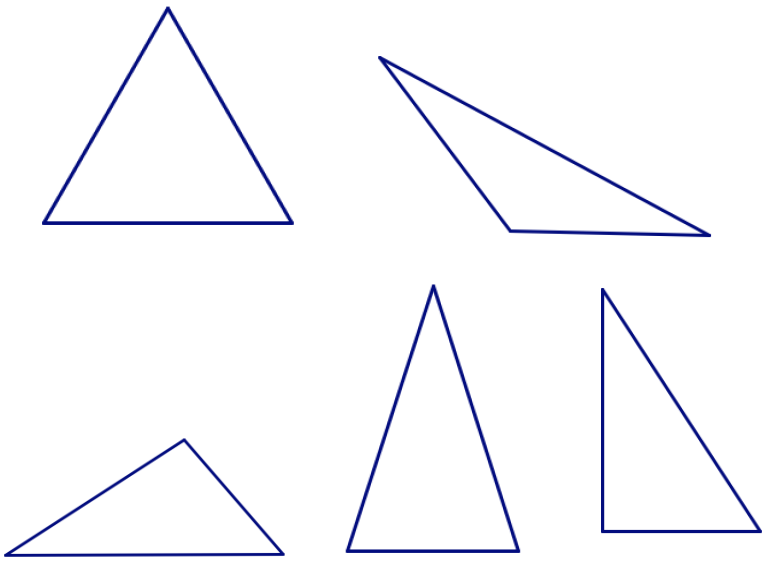
# Lattice Polygons

**From *Geometry: A Guide for Teachers***
***Mathematical Circles Library***
**by Judith and Paul Sally.**

Warm-up 1: How many ways can you make change for a dollar using only dimes, nickels, and pennies? Using dimes, nickels, pennies, and quarters?

Warm-up 2:

1. What is a right triangle? How do you find its area?
2. What is an equilateral triangle?
3. What is an isosceles triangle?
4. What is a scalene triangle?
5. What is a polygon?



A lattice point is a point where there is a peg on the geoboard, or a point where grid lines cross on graph paper, or a point marked with a dot on lattice paper.

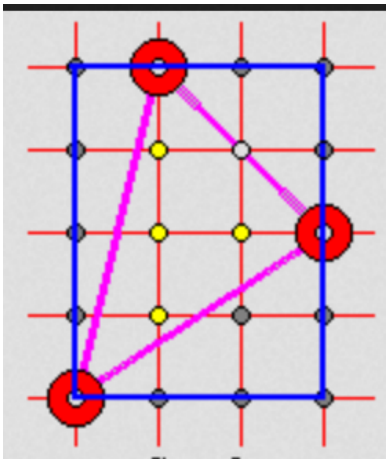
A lattice square is a square with all vertices at lattice points; a lattice triangle is a triangle with all vertices at lattice points. What do you think a lattice polygon is?

# 1 Lattice Triangles and Their Areas

1. Construct
   (a) a lattice right triangle
   (b) a lattice iscosceles triangle
   (c) a lattice scalene triangle
2. Construct a lattice right triangle with legs that are neither horizontal or vertical. How do you know it is a right triangle?
3. For each of your triangles, calculate the area. Hint: sometimes it is easier to calculate the area *outside* the triangle instead of the area *inside* the triangle.
4. For each triangle, count the number of pegs inside the triangle (call this $I$). Count the number of pegs that are on the boundary edges or vertices of the triangle (call this $B$). Fill in the chart.
5. Do you notice any patterns between area, $I$, and $B$?

| Triangle | Area | $I$ | $B$ |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |

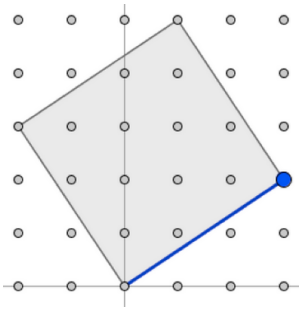
Example: $I = 4$, $B = 4$, Area $= 5$



6. Does the relationship you found work for other shapes besides triangles?
7. How many ways can you make change for a dollar using only dimes, nickels, and pennies? Using dimes, nickels, pennies, and quarters? Hint: chart out the options of dimes and nickels on lattice paper.

# 2 Lattice Squares

1. Assume each square of the graph paper has area 1. Draw a lattice square with
   (a) area 1
   (b) area 4
   (c) area 36
2. Is it possible to draw a lattice square with
   (a) area 2?
   (b) area 3?
   (c) area 5?
3. What is the area of this square?



4. What numbers are possible for the areas of lattice squares?

| Area | Possible? | Area | Possible? | Area | Possible? |
|---|---|---|---|---|---|
| 1 | | 11 | | 21 | |
| 2 | | 12 | | 22 | |
| 3 | | 13 | | 23 | |
| 4 | | 14 | | 24 | |
| 5 | | 15 | | 25 | |
| 6 | | 16 | | 26 | |
| 7 | | 17 | | 27 | |
| 8 | | 18 | | 28 | |
| 9 | | 19 | | 29 | |
| 10 | | 20 | | 30 | |