2015
AMC 8

DO NOT OPEN UNTIL TUESDAY, NOVEMBER 17, 2015

****ADMINISTRATION ON AN EARLIER DATE WILL DISQUALIFY YOUR SCHOOL'S RESULTS****

1. PLEASE READ THE TEACHERS' MANUAL BEFORE NOVEMBER 17, 2015. All rules and instructions needed to administer this exam are contained in the manual. You will not need anything from inside this package until November 17.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 8 CERTIFICATION FORM that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be sent by trackable mail to the AMC office no later than 24 hours following the exam.
4. THE AMC 8 IS TO BE ADMINISTERED DURING A CONVENIENT 40 MINUTE PERIOD. THE EXAM MAY BE GIVEN DURING A REGULAR MATH CLASS.
5. The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, email, internet or media of any type during this period is a violation of the competition rules.

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CELEBRATING A CENTURY OF ADVANCING MATHEMATICS

MAA American Mathematics Competitions

31st Annual

AMC 8

American Mathematics Contest 8

Tuesday, November 17, 2015

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. For each question, only one answer choice is correct.
3. Mark your answer to each problem on the AMC 8 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. There is no penalty for guessing. Your score is the number of correct answers.
5. Only scratch paper, graph paper, rulers, protractors, and erasers are allowed as aids. Calculators are NOT allowed. No problems on the test *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record your information on the answer form.
8. You will have 40 minutes to complete the test once your proctor tells you to begin.
9. When you finish the exam, *sign your name* in the space provided on the answer form.

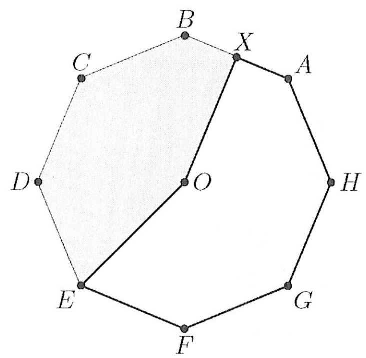
The Committee on the American Mathematics Competitions reserves the right to re-examine students before deciding whether to grant official status to their scores. The Committee also reserves the right to disqualify all scores from a school if it determines that the required security procedures were not followed.

1. How many square yards of carpet are required to cover a rectangular floor that is 12 feet long and 9 feet wide? (There are 3 feet in a yard.)

   (A) 12 **(B)** 36 **(C)** 108 **(D)** 324 **(E)** 972

2. Point O is the center of the regular octagon $ABCDEFGH$, and X is the midpoint of side $\overline{AB}$. What fraction of the area of the octagon is shaded?

   (A) $\frac{11}{32}$ **(B)** $\frac{3}{8}$ **(C)** $\frac{13}{32}$ **(D)** $\frac{7}{16}$ **(E)** $\frac{15}{32}$

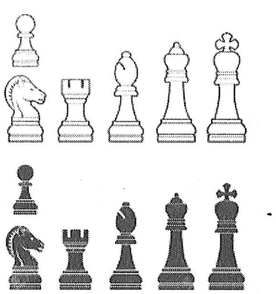


3. Jack and Jill are going swimming at a pool that is one mile from their house. They leave home simultaneously. Jill rides her bicycle to the pool at a constant speed of 10 miles per hour. Jack walks to the pool at a constant speed of 4 miles per hour. How many minutes before Jack does Jill arrive?

   (A) 5 **(B)** 6 **(C)** 8 **(D)** 9 **(E)** 10

4. The Centerville Middle School chess team consists of two boys and three girls. A photographer wants to take a picture of the team to appear in the local newspaper. She decides to have them sit in a row with a boy at each end and the three girls in the middle. How many such arrangements are possible?

   (A) 2 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 12

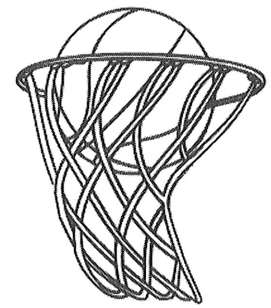


5. Billy's basketball team scored the following points over the course of the first 11 games of the season:

   $$42, 47, 53, 53, 58, 58, 58, 61, 64, 65, 73.$$

   If his team scores 40 in the 12th game, which of the following statistics will show an increase?

   (A) range **(B)** median **(C)** mean **(D)** mode **(E)** mid-range



6. In $\triangle ABC$, $AB = BC = 29$, and $AC = 42$. What is the area of $\triangle ABC$?

   (A) 100 **(B)** 420 **(C)** 500 **(D)** 609 **(E)** 701

7. Each of two boxes contains three chips numbered 1, 2, 3. A chip is drawn randomly from each box and the numbers on the two chips are multiplied. What is the probability that their product is even?

   (A) $\frac{1}{9}$ **(B)** $\frac{2}{9}$ **(C)** $\frac{4}{9}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{5}{9}$

8. What is the smallest whole number larger than the perimeter of any triangle with a side of length 5 and a side of length 19?

   (A) 24 **(B)** 29 **(C)** 43 **(D)** 48 **(E)** 57

9. On her first day of work, Janabel sold one widget. On day two, she sold three widgets. On day three, she sold five widgets, and on each succeeding day, she sold two more widgets than she had sold on the previous day. How many widgets in total had Janabel sold after working 20 days?

   (A) 39 **(B)** 40 **(C)** 210 **(D)** 400 **(E)** 401

10. How many integers between 1000 and 9999 have four distinct digits?

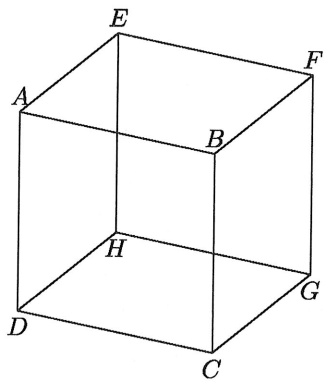
    (A) 3024 **(B)** 4536 **(C)** 5040 **(D)** 6480 **(E)** 6561

11. In the small country of Mathland, all automobile license plates have four symbols. The first must be a vowel (A, E, I, O, or U), the second and third must be two different letters among the 21 non-vowels, and the fourth must be a digit (0 through 9). If the symbols are chosen at random subject to these conditions, what is the probability that the plate will read "AMC8"?

(A) $\frac{1}{22{,}050}$ **(B)** $\frac{1}{21{,}000}$ **(C)** $\frac{1}{10{,}500}$ **(D)** $\frac{1}{2{,}100}$ **(E)** $\frac{1}{1{,}050}$

12. How many pairs of parallel edges, such as $\overline{AB}$ and $\overline{GH}$ or $\overline{EH}$ and $\overline{FG}$, does a cube have?

(A) 6 **(B)** 12 **(C)** 18 **(D)** 24 **(E)** 36



13. How many subsets of two elements can be removed from the set

$$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$$

so that the mean (average) of the nine remaining numbers is 6?

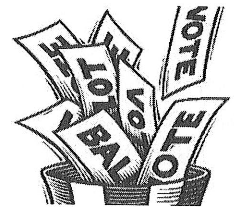
(A) 1 **(B)** 2 **(C)** 3 **(D)** 5 **(E)** 6

14. Which of the following integers cannot be written as the sum of four consecutive odd integers?

(A) 16 **(B)** 40 **(C)** 72 **(D)** 100 **(E)** 200

15. At Euler Middle School, 198 students voted on two issues in a school referendum with the following results: 149 voted in favor of the first issue and 119 voted in favor of the second issue. If there were exactly 29 students who voted against both issues, how many students voted in favor of both issues?

(A) 49 **(B)** 70 **(C)** 79 **(D)** 99 **(E)** 149



16. In a middle-school mentoring program, a number of the sixth graders are paired with a ninth-grade student as a buddy. No ninth grader is assigned more than one sixth-grade buddy. If $\frac{1}{3}$ of all the ninth graders are paired with $\frac{2}{5}$ of all the sixth graders, what fraction of the total number of sixth and ninth graders have a buddy?

(A) $\frac{2}{15}$ **(B)** $\frac{4}{11}$ **(C)** $\frac{11}{30}$ **(D)** $\frac{3}{8}$ **(E)** $\frac{11}{15}$

17. Jeremy's father drives him to school in rush hour traffic in 20 minutes. One day there is no traffic, so his father can drive him 18 miles per hour faster and gets him to school in 12 minutes. How far in miles is it to school?

(A) 4 **(B)** 6 **(C)** 8 **(D)** 9 **(E)** 12

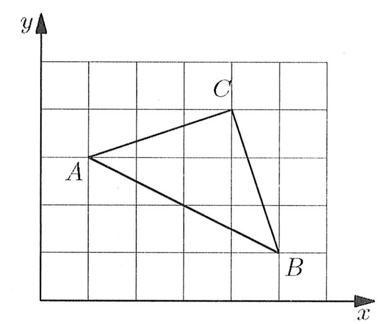
18. An arithmetic sequence is a sequence in which each term after the first is obtained by adding a constant to the previous term. For example, 2, 5, 8, 11, 14 is an arithmetic sequence with five terms, in which the first term is 2 and the constant added is 3. Each row and each column in this 5×5 array is an arithmetic sequence with five terms. What is the value of X?

(A) 21 **(B)** 31 **(C)** 36 **(D)** 40 **(E)** 42

1				25
		X		
17				81

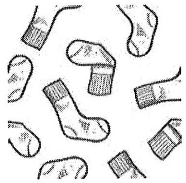
19. A triangle with vertices at $A = (1, 3)$, $B = (5, 1)$, and $C = (4, 4)$ is plotted on a 6×5 grid. What fraction of the grid is covered by the triangle?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$



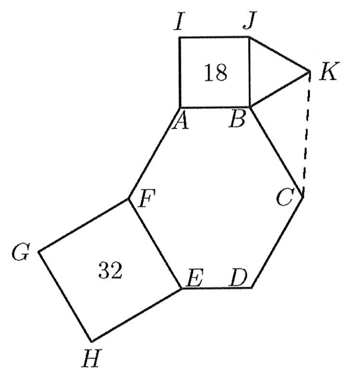
20. Ralph went to the store and bought 12 pairs of socks for a total of \$24. Some of the socks he bought cost \$1 a pair, some of the socks he bought cost \$3 a pair, and some of the socks he bought cost \$4 a pair. If he bought at least one pair of each type, how many pairs of \$1 socks did Ralph buy?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8



21. In the given figure hexagon $ABCDEF$ is equiangular, $ABJI$ and $FEHG$ are squares with areas 18 and 32 respectively, $\triangle JBK$ is equilateral and $FE = BC$. What is the area of $\triangle KBC$?

(A) $6\sqrt{2}$ **(B)** 9 **(C)** 12 **(D)** $9\sqrt{2}$ **(E)** 32



22. On June 1, a group of students is standing in rows, with 15 students in each row. On June 2, the same group is standing with all of the students in one long row. On June 3, the same group is standing with just one student in each row. On June 4, the same group is standing with 6 students in each row. This process continues through June 12 with a different number of students per row each day. However, on June 13, they cannot find a new way of organizing the students. What is the smallest possible number of students in the group?

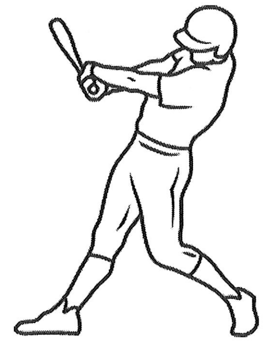
(A) 21 **(B)** 30 **(C)** 60 **(D)** 90 **(E)** 1080

23. Tom has twelve slips of paper which he wants to put into five cups labeled A, B, C, D, E. He wants the sum of the numbers on the slips in each cup to be an integer. Furthermore, he wants the five integers to be consecutive and increasing from A to E. The numbers on the papers are 2, 2, 2, 2.5, 2.5, 3, 3, 3, 3, 3.5, 4, and 4.5. If a slip with 2 goes into cup E and a slip with 3 goes into cup B, then the slip with 3.5 must go into what cup?

(A) A **(B)** B **(C)** C **(D)** D **(E)** E

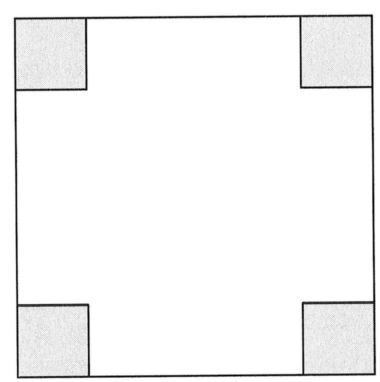
24. A baseball league consists of two four-team divisions. Each team plays every other team in its division N games. Each team plays every team in the other division M games with $N > 2M$ and $M > 4$. Each team plays a 76 game schedule. How many games does a team play within its own division?

(A) 36 **(B)** 48 **(C)** 54 **(D)** 60 **(E)** 72



25. One-inch squares are cut from the corners of this 5 inch square. What is the area in square inches of the largest square that can be fitted into the remaining space?

(A) 9 **(B)** $12\frac{1}{2}$ **(C)** 15 **(D)** $15\frac{1}{2}$ **(E)** 17

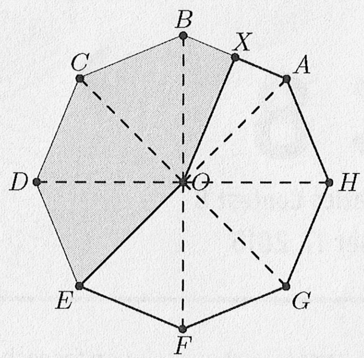


1. **Answer (A):** The floor is $\frac{12}{3} = 4$ yards long and $\frac{9}{3} = 3$ yards wide, so it will take $4 \times 3 = 12$ square yards of carpet to cover it.

OR

The area of the floor is 12×9 square feet. There are $3^2 = 9$ square feet in a square yard, so the number of square yards required is $\frac{12 \times 9}{9} = 12$.

2. **Answer (D):** The octagon can be divided into 8 congruent triangles, 3 of which are $\triangle BOC$, $\triangle COD$, and $\triangle DOE$. The area of $\triangle XOB$ is half the area of one of these, so the fraction of the area of the octagon that is shaded is $3 \cdot \frac{1}{8} + \frac{1}{2} \cdot \frac{1}{8} = \frac{7}{16}$.



3. **Answer (D):** Jill takes $\frac{1}{10}$ of an hour, or 6 minutes, to get to the pool, and Jack takes $\frac{1}{4}$ of an hour, or 15 minutes, so Jill arrives $15 - 6 = 9$ minutes before Jack.

4. **Answer (E):** There are 2 ways to seat the boys, one on each end, and $3 \cdot 2 \cdot 1 = 6$ ways to seat the three girls in the middle. So there are $2 \cdot 6 = 12$ possible arrangements.

5. **Answer (A):** The range is the high score minus the low score, so the range changes from 31 to 33. The range is the only listed statistic that will increase. Because 40 is the lowest score for the season, it will cause the mean to decrease. The median value of the first 11 games is the 6th highest score, or 58. The median value of the first 12 games will be the average of the 6th highest and 7th highest scores, or $(58 + 58)/2 = 58$, so no change will occur in the median. Similarly, the score that occurs most frequently in either situation is 58, so the mode will not change. The mid-range is the average of the highest score and the lowest score. The mid-range of the first 11 games is $(73 + 42)/2 = 57.5$. The mid-range of the first 12 games is 56.5, a decrease from 57.5.

6. **Answer (B):** Let D be the midpoint of side $\overline{AC}$. Then $\overline{BD}$ is the altitude to $\overline{AC}$ and $\triangle BDC$ is a right triangle with $BC = 29$ and $DC = 21$. So $BD = \sqrt{29^2 - 21^2} = \sqrt{400} = 20$. The area of $\triangle ABC = \frac{1}{2} \cdot 20 \cdot 42 = 420$.

OR

Heron's formula gives the area of a triangle in terms of the lengths of its sides. If the side lengths are a, b and c, then let $s = \frac{a+b+c}{2}$. The area is then $\sqrt{s(s-a)(s-b)(s-c)}$. In this problem, $s = \frac{29+29+42}{2} = 50$, and the area is $\sqrt{50 \cdot 21 \cdot 21 \cdot 8} = 21\sqrt{400} = 21 \cdot 20 = 420$.

7. **Answer (E):** The nine possible equally likely outcomes are:

$$(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)$$

In five of the nine outcomes the product is even. Therefore the probability is $\frac{5}{9}$.

OR

The only way the product of the two values could be odd is if an odd number is drawn from each box. The probability that this occurs is $\frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9}$. So the probability that the product is even is $1 - \frac{4}{9} = \frac{5}{9}$.

8. **Answer (D):** Let t be the length of the third side of the triangle. By the Triangle Inequality, $t < 5 + 19 = 24$. So the perimeter $5 + 19 + t < 5 + 19 + (5 + 19) = 48$.

9. **Answer (D):** Note that Janabel has sold a total of 1 widget after 1 day, $1 + 3 = 4 = 2^2$ after 2 days and $1 + 3 + 5 = 3^2$ widgets after 3 days. It can be shown that this pattern continues so that after 20 days, Janabel has sold a total of $20^2 = 400$.

OR

The sum of the first 20 odd numbers $1, 3, 5, \ldots, 39$ is needed. The sum of the first and last is 40, the sum of the second and 19th is also 40, and in fact, there are 10 pairs of numbers that each add up to 40. Thus the required sum is 400.

10. **Answer (B):** The thousands position can be filled by the digits 1 through 9 (0 is excluded). Without repetition, the hundreds position can be filled with any of the remaining 9 digits (including 0). Similarly without repetition, the tens position and ones position can be filled with the remaining 8 and 7 digits, respectively. Thus there are $9 \cdot 9 \cdot 8 \cdot 7 = 4536$ integers between 1000 and 9999 that have distinct digits.

11. **Answer (B):** The first symbol can be any of the 5 vowels, the second can be any of the 21 consonants, the third can be any of the 20 other consonants, and the fourth can be any of the 10 digits. The total number of possible license plates is $5 \cdot 21 \cdot 20 \cdot 10 = 21{,}000$. Only one plate will read "AMC8", so the probability is $\frac{1}{21{,}000}$.

12. **Answer (C):** Each of the 12 edges is parallel to 3 other edges giving 36 possible pairs of parallel edges. But each pair of parallel edges is counted twice in this process, so there are 18 pairs of parallel edges.

OR

There are 6 pairs of parallel edges related to $\overline{AB}$ ($\overline{AB} \| \overline{EF}$, $\overline{AB} \| \overline{HG}$,$\overline{AB} \| \overline{DC}$, $\overline{EF} \| \overline{HG}$, $\overline{EF} \| \overline{DC}$, $\overline{HG} \| \overline{DC}$). Similarly there are 6 pairs of parallel edges related to $\overline{AE}$ and 6 pairs of parallel edges related to $\overline{AD}$ for a total of 18 pairs of parallel edges.

13. **Answer (D):** If the average of the remaining 9 numbers is 6, then their sum is 54. Because the sum of the numbers in the original set is 66, the sum of the two numbers removed must be 12. There are five such subsets: $\{1, 11\}$, $\{2, 10\}$, $\{3, 9\}$, $\{4, 8\}$, and $\{5, 7\}$.

14. **Answer (D):** The sum of 4 consecutive odd integers is always a multiple of 8, $(2n-3)+(2n-1)+(2n+1)+(2n+3) = 8n$. Among the given choices, only 100 is not a multiple of 8. The other four numbers can each be written as the sum of four consecutive odd numbers:

$$
\begin{aligned}
16 &= 1 + 3 + 5 + 7 \\
40 &= 7 + 9 + 11 + 13 \\
72 &= 15 + 17 + 19 + 21 \\
200 &= 47 + 49 + 51 + 53
\end{aligned}
$$

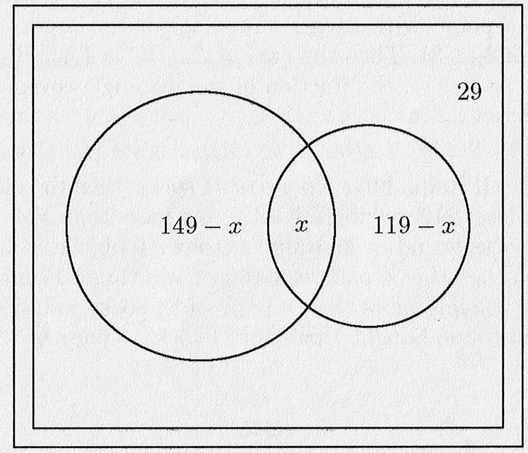
15. **Answer (D):** The sum $149 + 119 + 29 = 297$ counts the number of students who voted for both issues twice. So the number who voted in favor of both issues is $297 - 198$ or 99.

OR

In the diagram below, the left circle represents the 149 students who voted for the first issue, and the right circle represents the 119 students who voted for the second issue. Let x be the number of students who voted for both issues. Then $149 - x$ students voted for the first issue but not the second, $119 - x$ students

voted for the second issue but not the first and 29 students voted against both issues. The sum of the numbers in the diagram must be 198, so

$$
\begin{aligned}
(149 - x) + x + (119 - x) + 29 &= 198, \\
297 - x &= 198, \\
x &= 99.
\end{aligned}
$$



16. **Answer (B):** 2 out of every 5 sixth graders are paired with a ninth grade buddy, and 2 out of every 6 ninth graders are paired with a sixth grade buddy. (The ratios are now expressed so that the number of sixth graders matches the number of ninth graders.) So 4 out of every 11 students are in the mentoring program. The fraction is $\frac{4}{11}$.

OR

Suppose that n sixth graders are paired with n ninth graders. Then the total number of sixth graders is $\frac{5}{2}n$, the total number of ninth graders is $3n$, and the total number of sixth and ninth graders is $\frac{5}{2}n + 3n = \frac{11}{2}n$. There are $2n$ students in the mentoring program, which is $\dfrac{2n}{\frac{11}{2}n} = \dfrac{4}{11}$ of the total number of students.

17. **Answer (D):** Because the new time is $\frac{12}{20} = \frac{3}{5}$ of the original time, the new speed must be $\frac{5}{3} = 1\frac{2}{3}$ of the original speed. Then the additional 18 miles per hour must be $\frac{2}{3}$ of the original speed, which is then 27 mph. In 20 minutes, Jeremy's father travels $\frac{1}{3} \cdot 27 = 9$ miles.

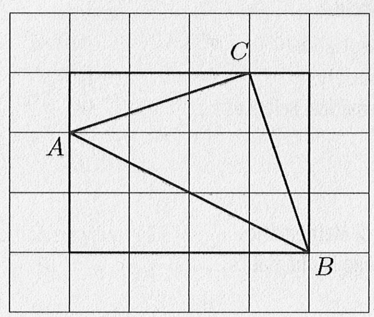
OR

Let r be Jeremy's original speed in miles per hour. Twenty minutes is $\frac{20}{60} = \frac{1}{3}$ of an hour and twelve minutes is $\frac{12}{60} = \frac{1}{5}$ of an hour. Then $\frac{1}{3}r = \frac{1}{5}(r + 18)$, so $5r = 3r + 54$, and $r = 27$. Thus the distance to the school is $\frac{1}{3}r = 9$ miles.

18. **Answer (B):** The middle number in an arithmetic sequence with 5 terms is the average of the first and last numbers. The average of 1 and 25 is 13. The average of 17 and 81 is 49. Thus, X is the average of 13 and 49, or 31. Alternatively, $X = \frac{9+53}{2} = 31$. In fact X is the average of the four corner entries.

1		13		25
9		31		53
17		49		81

19. **Answer (A):** The triangle is inscribed in a 4×3 rectangle with vertices at $(1, 1)$, $(1, 4)$, $(5, 4)$, and $(5, 1)$. Three triangular regions are inside the 4×3 rectangle but outside $\triangle ABC$. The area of the lower-left triangle is $\frac{1}{2} \cdot 4 \cdot 2 = 4$ square units. The area of the upper-left triangle is $\frac{1}{2} \cdot 1 \cdot 3 = \frac{3}{2}$ square units. The area of the third triangle is also $\frac{1}{2} \cdot 1 \cdot 3 = \frac{3}{2}$ square units. So the area of $\triangle ABC$ is $12 - 4 - \frac{3}{2} - \frac{3}{2} = 5$ square units. The area of the 6×5 grid is 30 square units. Thus, the fraction covered by the triangle is $\frac{5}{30} = \frac{1}{6}$.



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OR

Pick's Theorem says that the area of a polygonal region whose vertices are at lattice points (points whose coordinates are integers) is given by $A = I + \frac{1}{2}B - 1$ where I is the number of lattice points in the interior of the region and B is the number of lattice points on the boundary. Referring to the figure above, there are $B = 4$ lattice points on the boundary of $\triangle ABC$ at $(1,3)$, $(3,2)$, $(5,1)$, and $(4,4)$. There are $I = 4$ points with integer coordinates in the interior of $\triangle ABC$ at $(2,3)$, $(3,3)$, $(4,2)$, and $(4,3)$. Then the area of $\triangle ABC$ is $I + \frac{1}{2}B - 1 = 4 + 2 - 1 = 5$ square units. As before, the fraction of the rectangle covered by the triangle is $\frac{5}{30} = \frac{1}{6}$.

20. **Answer (D):** If Ralph buys 6 pairs of \$1 socks, then the other 6 pairs of socks would cost at least \$19 making the total cost more than \$24. Buying fewer than 6 pairs of \$1 socks would make Ralph's cost even higher. If he bought 8 pairs of \$1 socks, then the other 4 pairs would cost less than \$16 making the total cost less than \$24. Buying more than 8 pairs of \$1 socks would make his total cost even lower. So Ralph bought 7 pairs of \$1 socks, 3 pairs of \$3 socks, and 2 pairs of \$4 socks.

OR

Let a, b and c be the number of pairs of \$1, \$3 and \$4 socks, respectively. Then $a + b + c = 12$ and $a + 3b + 4c = 24$. Subtracting the first equation from the second gives $2b + 3c = 12$. Since 3 is a factor of both 12 and $3c$, 3 must also be a factor of $2b$. Since $c > 0$, it follows that $b = 3$, $c = 2$, and $a = 7$.

21. **Answer (C):** The area of the square $ABJI$ is 18 and $\triangle KJB$ is equilateral, so $KB = JB = \sqrt{18} = 3\sqrt{2}$. The area of the square $FEHG$ is 32, so $BC = FE = \sqrt{32} = 4\sqrt{2}$. Each interior angle of the hexagon is $120°$, so $\angle KBC = 360° - 60° - 90° - 120° = 90°$ and $\triangle KBC$ is a right triangle. Its area is $\frac{1}{2} \cdot 3\sqrt{2} \cdot 4\sqrt{2} = 12$.

22. **Answer (C):** The number of students must be a multiple of 6 and also a multiple of 15. So the number of students must be divisible by the least common multiple of 6 and 15 which is 30. The divisors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30, so there are only 8 divisors. The divisors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30 and 60. So 60 has 12 divisors and 60 is the smallest possible number of students.

23. **Answer (D):**

The sum of the numbers is 35. So the 5 consecutive numbers in the cups must be 5, 6, 7, 8, and 9. It is impossible to get a sum of 5 or 7 using the slip with

3.5. Cup B needs a sum of 6, but it already has a slip with 3 on it so the slip with a 3.5 can't go there. Cup E needs a sum of 9, but with a slip with 2 in it the slip with 3.5 can't go there. The only place the slip with 3.5 on it can go is Cup D. One possibility is:

Cup A. 2, 3
Cup B. 3, 3
Cup C. 2.5, 4.5
Cup D. 2, 2.5, 3.5
Cup E. 2, 3, 4

24. **Answer (B):** The number of games played by a team is $3N + 4M = 76$. Because $M > 4$ and $N > 2M$ it follows that $N > 8$. Because 4 divides both 76 and $4M$, 4 must divide $3N$ and hence N. If $N = 12$ then $M = 10$ and the condition $N > 2M$ is not satisfied. If $N \geq 20$ then $M \leq 4$ and the condition $M > 4$ is not satisfied. So the only possibility is $N = 16$ and $M = 7$. So each team plays $3 \cdot 16 = 48$ games within its division and $4 \cdot 7 = 28$ games against the other division.

OR

The total number of games played by each team is $3N + 4M = 76$. Make a chart of possibilities with $M > 4$:

M	$4M$	$76 - 4M = 3N$	N
5	20	56	(not an integer)
6	24	52	(not an integer)
7	28	48	16
8	32	44	(not an integer)
9	36	40	(not an integer)
10	40	36	12
11	44	32	(not an integer)
12	48	28	(not an integer)
13	52	24	8

Only $M = 7$ and $N = 16$ satisfy the conditions.

The case $M = 10$ and $N = 12$ violates the condition $N > 2M$.

So each team plays $3N = 48$ divisional games and $4M = 28$ games against the other division.

This is modeled on the Pioneer Baseball League with teams in Colorado, Idaho, Montana, and Utah.

25. **Answer (C):** Let $EQ = c$ and $TQ = s$ as indicated in the figure. Triangles QUV and FEQ are similar since $\angle FQE$ and $\angle QVU$ are congruent because

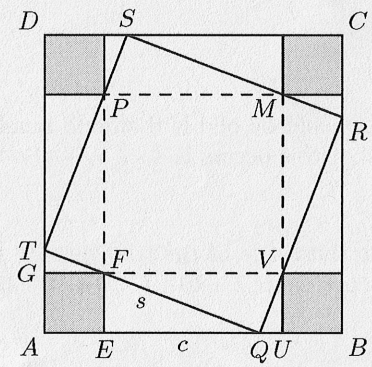
both are complementary to $\angle VQU$. So

$$\frac{QU}{UV} = \frac{FE}{EQ}$$

and thus $QU = \frac{1}{c}$. Then $AB = 1 + c + 1/c + 1 = 5$ and so $c + 1/c = 3$. Since the area of square $ABCD$ equals the sum of areas of square $QRST$, four unit squares, four $1 \times c$ triangles, and four $\frac{1}{c} \times 1$ triangles, it follows that

$$\begin{aligned} 25 &= s^2 + 4\left(1 + \frac{c}{2} + \frac{1}{2c}\right) \\ &= s^2 + 4 + 2\left(c + \frac{1}{c}\right) \\ &= s^2 + 4 + 2 \cdot 3 \end{aligned}$$

Therefore, the area of square $QRST = s^2 = 15$.



OR

Square $FVMP$ has area 9, the four triangles FQV, VRM, MSP, and PTF each have area $\frac{3}{2}$. So the area of square $STQR$ is $9 + 6 = 15$.