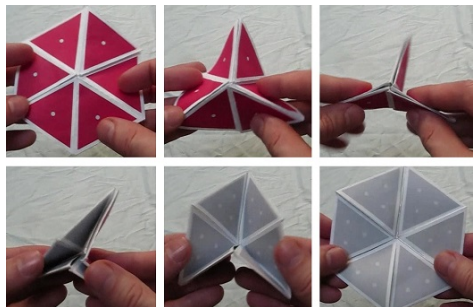


Hexaflexagons¹

1. Pick two different colors and color each side of your hexaflexagon a different color.
2. Flex the hexaflexagon by pinching it and gently pulling the center out, to uncover a third, uncolored side. Color it with a new color.

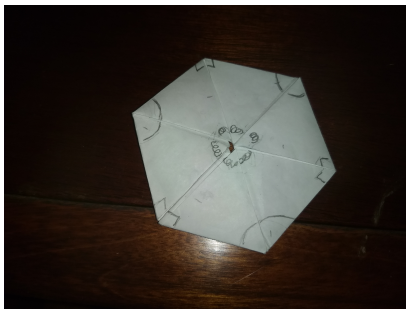


3. Continue flexing the hexaflexagon. Can you uncover any additional sides? How many sides, total, can you find? Color each new side with a color you haven't used yet.
4. Once you have colored all the sides you can find, pair up with a neighbor and compare results. Do the rest of these instructions together, working on one hexaflexagon.
5. Make a chart of which colors go together front and back. It might help to put numbers on the sides. For example, side 1 might sometimes be on the back of side 2, and other times on the back of side 3. Which colors / numbers seem to come up most frequently? Which are hard to find?

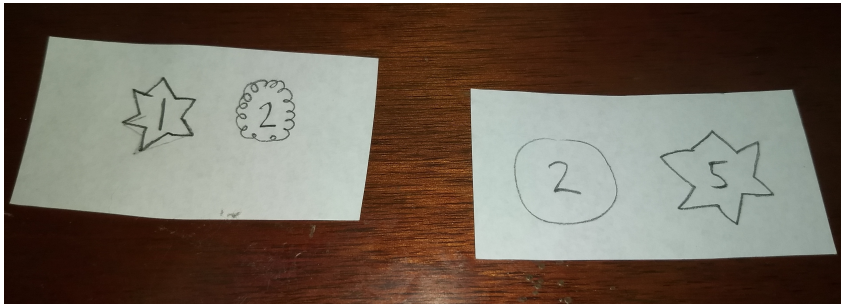
Side number	Which other sides appear on the back of this side?
1	
2	
3	
4	
5	
6	

¹See Vi Hart videos to learn how to make these

6. Some sides can be flexed in two different ways, depending on how you rotate your fingers before flexing. Which sides can be flexed in two ways and which in just one?
7. Draw a star in the center of one of the sides that comes up frequently. As you flex, can you get to a position where the star is now on the outside, not the inside? If so, draw another shape on the inside, like a circle, or a squiggly line.
8. Every time you have find center that is not decorated with a star, circle, or curly line, draw one. Make sure you use different decorations for different "centers" on the same side – if you are drawing on the yellow side and you already drew a circle in the center that migrated to the outside, next time use a star, and next time use a squiggly line for the center of that side.



9. Let's try to figure out how all the sides fit together. Write down pairs of sides with both numbers and decorated centers that go front and back from each other. For example, 1 with a star might go front-to-back with 2 with a squiggly line, and 2 with a circle line might go front to back with 5 with a star. Draw the symbols on paper rectangles to make dominoes.



10. Now fit together the dominoes so that 5 goes next to 5, 4 goes next to 4, etc. It's ok if some of the numbers are upside down. Try to fit them around in triples. This gives you a map of how to get to all the sides of the hexaflexagon!
11. Use your map to get to the hard-to-find sides efficiently!
12. Hexaflexagons are made out of strips of paper folded into lots of triangles. How many triangles would you need on a strip to make one like this? Remember that a regular sheet of paper already has two sides. This one is called a hexa-hexaflexagon, because it has 6 sides and each side looks like a hexagon.

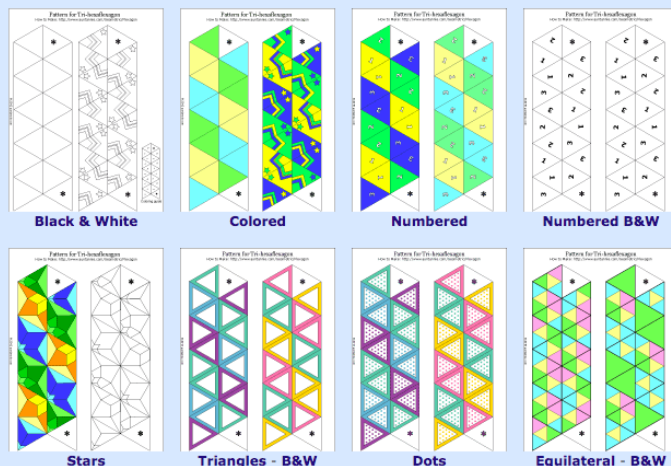
13. A tri-hexaflexagon has 3 sides, and each side looks like a hexagon. How many triangles would you need to make a tri-hexaflexagon?
14. Use the instructions and templates to make your own tri-hexaflexagon or hexa-hexaflexagon.

Step 1: Choose a Pattern and Print It

Choose a tri-hexaflexagon to make. Download and print the pattern. Choose a design that prints in color, or select a black & white (B&W) pattern and use your creativity to add color and designs. You might want to start with the pattern with numbers on each face—1, 2 and 3. The **Dots** pattern coordinates with the dot patterns in the **Geometric Solids** craft project.

On the Stars pattern, half of the fold lines are marked on each side, so you will need to score on both sides. It's best to make another pattern first for experience in scoring and folding.

On the Equilateral triangle patterns, the fold lines are thicker than the lines for the individual, smaller triangles. Select the black & white (B&W) Equilateral pattern to color the triangles as you choose.



Step 2: Color Pattern (optional)

Color the pattern template as you like with markers, colored pencils or crayons. You don't need to color the glue tabs marked with '*'.

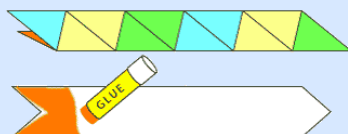
Step 3: Cut

Cut around the pattern on the outline. Try to make the cuts very straight and accurate. There will be one piece with twenty triangles.

Step 4: Double

Make a double thickness by folding the pattern piece in half lengthwise and glue. To make a sharp, straight fold, first score on the fold line.

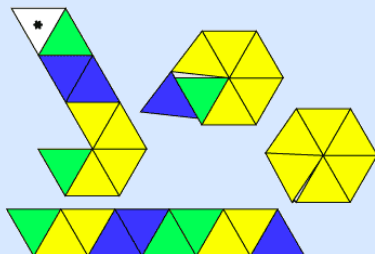
To score: Use a ruler and the empty ballpoint pen (or bone folder) to make an indent along the fold lines.



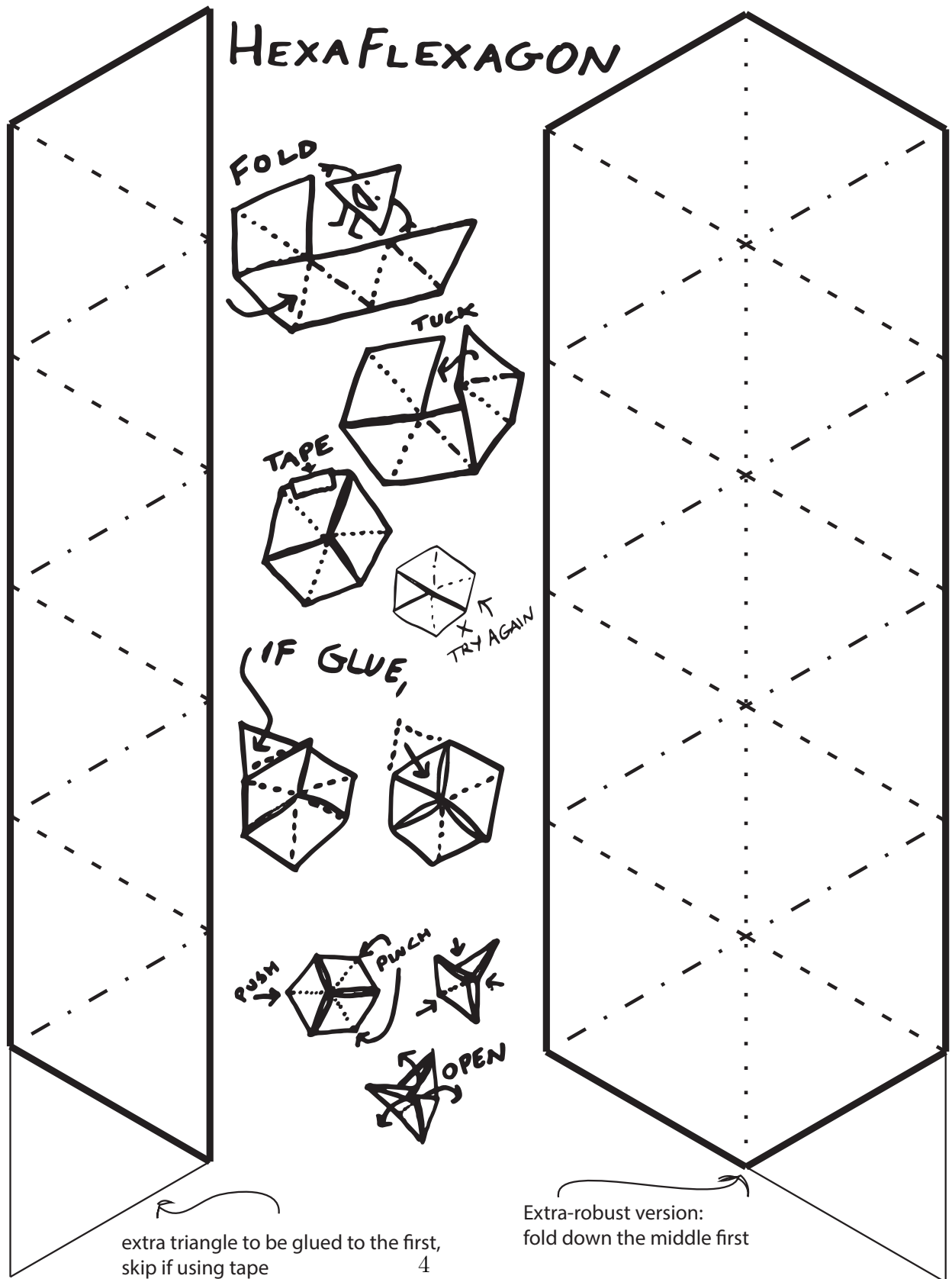
Step 5: Fold

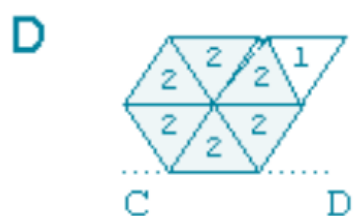
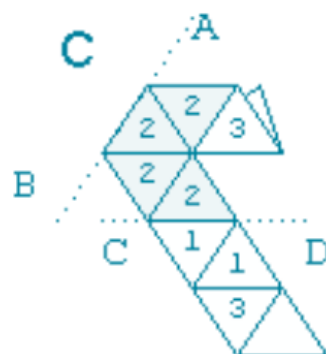
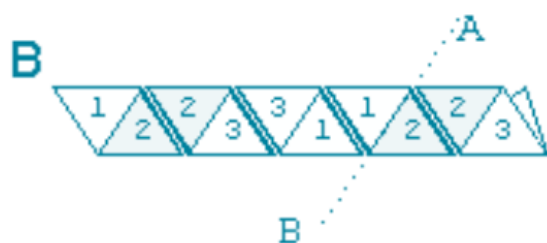
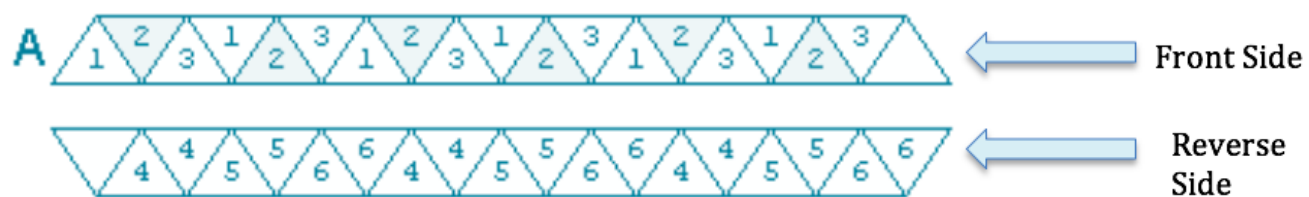
Score and fold back and forth on all the solid lines. Do this for each of the 10 triangles of the tri-hexaflexagon.

Place the folded pattern with the side without glue tabs facing up, and the end with a green (2) triangle to the left. Fold back on the line between the yellow (3rd) and blue (4th) triangles (1 and 3)—four yellow triangles (1s) are together now. Fold back on the line between the yellow and blue triangles (1 and 3). Now you should be able to see five yellow triangles (1s). Pull the yellow triangle (1) forward that is behind the green triangle (2). There should be six yellow triangles (1s) on top and a blue triangle (3) to the side. Fold the blue triangle (3) back.



The two glue tabs marked with '*' are now face-to-face. Glue them together. Wait for the glue to dry before flexing your flexagon.







Julia Robinson
Mathematics Festival

Hexaflexagons



Challenges

1. Use the instructions on the back to make a hexaflexagon with six faces.
2. Flex the hexaflexagon so you can see different faces on front and back. Can you find every face this way? Are there any faces you cannot see?
3. Pick a face. Starting from that face, how many flexes do you need to do in order to see each face at least once? Is this the same for all faces?
4. Does your answer change depending on whether or not you're allowed to turn your hexaflexagon over before flexing it to switch between faces?
5. Do you think it would be useful to draw a picture to show how you can move from face to face? How would you draw this picture?
6. How can you make a tri-hexaflexagon, a flexagon that has only 3 different faces? How many triangles do you need to begin with?

