Mathematical Games, Part 2

1. **Sum to 23** This game begins with the number 0. In one turn, a player can add 1, 2, or 4, but can't go over 23. The player who reaches exactly 23 wins.
2. **Candy Box** A box contains 300 gummy bears. Players take turns removing no more than half the gummy bears in the box. The player who cannot move loses.
3. **Triangles** Trace 6 dots around a circle. Each player (of 2) should have a different color pencil. Take turns drawing line segments. Each line segment connects 2 dots. The way to not lose is to avoid drawing 3 lines in your color that connect to make a triangle. If you do make a triangle in your color, you lose. Try to force your opponent into having to draw a triangle in her color. If someone completes a triangle, but not all 3 sides are in the same color, it doesn't count. The losing triangle must not only be all 3 sides in the same color, but also, all of the vertices must be the original dots.
4. **Two Pile Nim.**
   (a) Now there are two piles of pennies, one pile with 10 pennies and another one with 7. On each turn, a player can take any number of pennies from either one of the two piles. The player unable to move (no pennies left) loses.
   (b) What about if the numbers of pennies in the piles are m and n?
5. **Inchworm** There are two piles of pennies; one pile contains 10 pennies while the other contains 7. A player can take one penny from the first pile, or one penny from the second pile, or one penny from each of the two piles. The player unable to move loses.
6. **Sneaky Inchworm** In addition to the moves described above, players are allowed to take a penny from the first pile and place it on the second pile or vice versa.
7. **Puppies and Kittens.** There are two piles of pennies; one pile contains 10 and one contains 7. A player can take any number of pennies from the first pile (the puppies), or any number from the second pile (the kittens), or the player can take the same number of pennies from both piles. For example, a player could take 2 from the first pile, or 6 from the second pile, or 3 from each pile. The player unable to move loses.

Extra Problems

8. **Ducks in a Row** There are 15 pennies in a row. Players take turns removing 1, 2, or 3 consecutive pennies at a time. "Consecutive" means located next to each other. The player who cannot move (because there are no pennies left) loses.

9. **Dominos on a Board** Players take turns placing 1 domino on an 8×8 board. Each domino should be placed so as to fully cover two squares of the board. The dominos cannot overlap. The player who cannot place a domino loses.

More Extra Problems

10. **Three Pile Nim.** There are three piles of pennies; one pile with 6 pennies, a second pile with 5 pennies, and a third pile with 3 pennies. Two players take turns removing any number of pennies from any one of the three piles. The player unable to move loses.

11. **Break the Bar.** You have a rectangular chocolate bar that is 6 x 8 squares in size. At each step, a player takes one piece of the chocolate and breaks it in two along a single straight line bounded by the squares. For example, you can turn the original bar into a 6 x 2 piece and a 6 x 6 piece, and this latter piece can be turned into a 1 x 6 piece and a 5 x 6 piece. The player who cannot make any more breaks loses.

12. **Free a Square.** Two players take turns breaking a piece of chocolate consisting of 5×10 small squares. At each turn, they may break along the division lines of the squares. The player who first obtains a single square of chocolate wins.

13. **Free a Square - Misere Version.** Same rules, but the first player to free a square loses.

Some of these games are from the book Mathematical Circles: Russian Experience by D. Fomin, S. Genkin, and I. Itenberg.