Mathematical Games

1. **One, Two, Three, Four, Takeaway**

   (a) There are 25 pennies on a table. On each turn, a player can take any number of pennies between 1 and 4. A player loses if he or she unable to move (because there are no pennies left).

   (b) Same game as above but it starts with 24 pennies.

   (c) Try other starting numbers of pennies.

2. **One, Two, Four Takeaway** There are 25 pennies in a pile. A player can take 1, 2, or 4 pennies on each turn. A player loses if he or she cannot continue (no more pennies left). Try other starting numbers of pennies.

3. **Lame Rook** On the top right square of an 8 by 8 chessboard there is a rook that can move either to the left or down through any number of squares. Players take turns moving the rook. A player loses if he or she is unable to move the rook (because it is already on the bottom left square). Consider various other initial positions of the rook.

4. **Lame Queen** On the top right square of an 8 by 8 chessboard there is a *queen* that can move any number of squares to the left, or down, or along a diagonal to the left and downwards. Players take turns moving the queen. A player loses if he or she is unable to move the queen. Consider various other initial positions of the queen.

Extra Problems for future weeks:

5. **Matches** A box contains 300 matches. Players take turns removing no more than half the matches in the box. The player who cannot move loses.

6. **Sum to 23** This game begins with the number 0. In one turn, a player can add 1, 2, or 4. The player who reaches exactly 23 wins.

7. **Two Pile Nim.**

   (a) Now there are two piles of pennies, one pile with 10 pennies and another one with 7. On each turn, a player can take any number of pennies from either one of the two piles. The player unable to move (no pennies left) loses.

   (b) What about if the numbers of pennies in the piles are m and n?

8. **Puppies and Kittens.** There are two piles of pennies; one pile contains 10 and one contains 7. A player can take any number of pennies from the first pile (the puppies), or any number from the second pile (the kittens), or the player can take the same number of pennies from both piles. For example, a player could take 2 from the first pile, or 6 from the second pile, or 3 from each pile. The player unable to move loses.

9. **Either, Or, Both.** There are two piles of pennies; one pile contains 10 pennies while the other contains 7. A player can take one penny from the first pile, or one penny from the second pile, or one penny from each of the two piles. The player unable to move loses.

10. **Ducks in a Row** There are 15 pennies in a row. Players take turns removing 1, 2, or 3 consecutive pennies at a time. "Consecutive" means located next to each other. The player who cannot move (because there are no pennies left) loses.

11. **Dominos on a Board** Players take turns placing 1 dominos on an 8×8 board. Each domino should be placed so as to fully cover two squares of the board. The dominos cannot overlap. The player who cannot place a domino loses.

12. **Inchworm** There are two piles of 7 pennies each. Players take turns taking a single penny from one of the piles, or a penny from each pile. The player who cannot move loses.

13. **Sneaky Inchworm** In addition to the moves described above, players are allowed to take a penny from the first pile and place it on the second pile or

vice versa.

14. **Three Pile Nim.** There are three piles of pennies; one pile with 6 pennies, a second pile with 5 pennies, and a third pile with 3 pennies. Two players take turns removing any number of pennies from any one of the three piles. The player unable to move loses.

Some of these games are from the book Mathematical Circles: Russian Experience by D. Fomin, S. Genkin, and I. Itenberg.