Chapel Hill Math Circle – 2.23.19

Numerical Puzzles 2: Encrypted Problems

1. Suppose somebody shows you this addition problem:

$$\begin{array}{r} \text{ME} \\ +\ \text{M} \\ \hline \text{ASA} \end{array}$$

This problem looks strange since the numbers are not quite numbers. Instead, they have been encrypted according to some code. Each digit was replaced by a letter. The same letter always stands for the same digit, and different letters stand for different digits. Our goal is to reconstruct the original numbers.

Where should we start? When a puzzle-breaker starts working with a ciphered piece of text, they'll look for a weak spot: a symbol that is easiest to guess. We will do the same.

Let's notice that none of the addends in the hundreds column have any value in the hundreds place. Therefore, A is equal to the carry from the tens column. Since we are adding up two numbers, the carry can not be greater than 1. Hence, A equals 1:

$$\begin{array}{r} \text{ME} \\ +\ \text{M} \\ \hline \text{ASA} \end{array} \quad \rightarrow \quad \begin{array}{r} \text{ME} \\ +\ \text{M} \\ \hline 1\text{S}1 \end{array}$$

We can formulate the same argument using estimates: ASA is the sum of a two-digit number and a one-digit number. Since ASA cannot be greater than 200, its hundred digit, A, must be 1.

To make a guess about the value of the next letter, we should concentrate on the addition in the tens column. The only number in this column is M, and nothing is added to it. Yet, the tens digit of the sum is different from M: it is equal to S. This difference can be explained only by the presence of a carry from the gives us the hundreds digit of the sum. If M were less than 9, then M + 1 would never produce a carry. Therefore, M = 9 and S = 0. Now it is easy to figure out that E = 2, since we know the values of all other digits:

$$\begin{array}{r} \text{ME} \\ +\ \text{M} \\ \hline 1\text{S}1 \end{array} \quad \rightarrow \quad \begin{array}{r} 9\text{E} \\ +\ 9 \\ \hline 101 \end{array} \quad \rightarrow \quad \begin{array}{r} 92 \\ +\ 9 \\ \hline 101 \end{array}$$

2. Decrypt the problem in the figure. The same letter always stands for the same digit, and different letters stand for different digits.

$$\begin{array}{r} \text{SUP} \\ +\ \text{SPU} \\ \hline \text{UPS} \end{array}$$

3. Decode AT + AT + AT = BAT

4. Decrypt the problems:

a)
$$\begin{array}{r} \text{AHA} \\ +\ \text{H} \\ \hline \text{BEE} \end{array}$$

b)
$$\begin{array}{r} \text{BB} \\ +\ \text{A} \\ \hline \text{CCC} \end{array}$$

c)
$$\begin{array}{r} \text{ODD} \\ +\ \text{ODD} \\ \hline \text{UNDO} \end{array}$$

5. Explain why the puzzles below have no solution.

a)
$$\begin{array}{r} \text{KATHRIN} \\ +\ \text{BELLA} \\ \hline \text{FRIENDS} \end{array}$$

b)
$$\begin{array}{r} \text{BAT} \\ +\ \text{RAT} \\ \hline \text{CAT} \end{array}$$