The Game of Criss Cross [1]

1 Warm-up

1. If a bird starts out 6 feet above the surface of the water and goes 9 feet straight down, where does it end up?
2. What is $6 - 9$? What is $9 - 6$?
3. What is
   (a) $13 - 17$?
   (b) $5 - 9 + 6$?
   (c) $15 - 27 + 4$?

2 The Game of Criss Cross

4. The game:
   - The game board is a piece of paper with three dots drawn at the vertices of an imaginary equilateral triangle, and between 2 and 7 dots drawn inside this imaginary triangle.
   - Players take turns drawing a single straight line segment joining any two points, as long as the segment does not pass through any other points or segments already appearing on the game board.
   - The winner is the last player able to make a legal move.
5. With a partner, play at least two games of Criss Cross. Record
   - who won the game (first player or second)
   - the total number of dots ("vertices") on your game board, including the original 3
   - the number of line segments ("edges")
   - the number of regions that the line segments divide your piece of paper into ("faces"). Include the region on the outside.

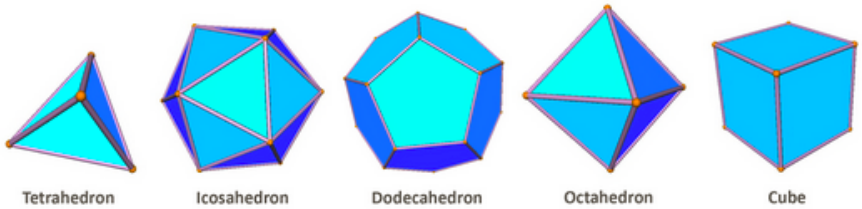
[1] From Sam Vandervelde *Circle in a Box*

6. Is there a winning strategy? Can you predict who will win the game?
7. Is there a relationship between the number of edges and the number of faces on a game board?
8. Is there a relationship between the number of vertices, the number of edges, and the number of faces?
9. How many edges will there be on a game board with 4 interior points (7 vertices total), after the game is finished? Who will win this game?
10. How many edges will there be on a game board with 7 interior points? Who will win this game? What about 100 interior points?

3 Polyhedra

11. Which "graph" of vertices, edges, and faces reminds you most of the tetrahedron? Why?

12. Match the graphs to the polyhedra that they most resemble.



13. What is $V - E + F$ for each of these polyhedra?
14. Is it possible to build a polyhedron out of exactly 5 triangles? Exactly 6 triangles? Exactly 7 triangles? Exactly 8 triangles?
15. Is it possible to build a polyhedron out of 12 triangles with exactly 4 triangles around each vertex?
16. Play and analyze some games of criss cross where the game board has 4 points at the corners of an imaginary square. Do the same relationships hold?