Pouring Water

1. Given a 5-liter unmarked container, a 3-liter unmarked container, and an unlimited supply of water, can you obtain an accurate measure of 4 liters of water?

   You can keep track of your pourings here.

5-liter	3-liter
0	0
5	0

5-liter	3-liter
0	0
0	3

   Could you also obtain an accurate measure of 1 L of water? 2 L?

2. Given a 4-liter unmarked container, a 7-liter unmarked container, and an unlimited supply of water, can you obtain an accurate measure of 5 liters of water? If so, what is the minimum number of pourings necessary?

4-liter	7-liter
0	0
4	0

4-liter	7-liter
0	0
0	7

   Could you also obtain an accurate measure of 1 L of water? 2 L? 3 L? 6 L?

3. Under the same conditions, but with a 3-liter container and a 6-liter container, can you obtain a measure of 5 liters? If so, what is the minimum number of pourings necessary?

3-liter	6-liter
0	0
3	0

3-liter	6-liter
0	0
0	6

Could you also obtain an accurate measure of 1 L of water? 2 L? 4 L?

4. Given a 5-liter unmarked container, a 9-liter unmarked container, and an unlimited supply of water, can you obtain an accurate measure of 6 liters of water? If so, what is the minimum number of pourings necessary?

5-liter	9-liter
0	0
5	0

5-liter	9-liter
0	0
0	9

Could you also obtain an accurate measure of 1 L of water? 2 L? 3 L? 4 L? 7 L ? 8 L ?

5. Given a 4-liter unmarked container, a 10-liter unmarked container, and an unlimited supply of water, can you obtain an accurate measure of 3 liters of water? If so, what is the minimum number of pourings necessary?

4-liter	10-liter
0	0
5	0

4-liter	10-liter
0	0
0	9

What numbers of liters of water can you obtain?

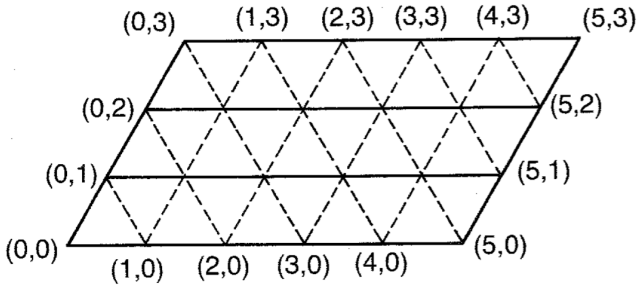
6. Given a 6-liter unmarked container, a 15-liter unmarked container, and an unlimited supply of water, can you obtain an accurate measure of 4 liters of water? If so, what is the minimum number of pourings necessary?

6-liter	15-liter
0	0
6	0

6-liter	15-liter
0	0
0	15

What numbers of liters of water can you obtain?

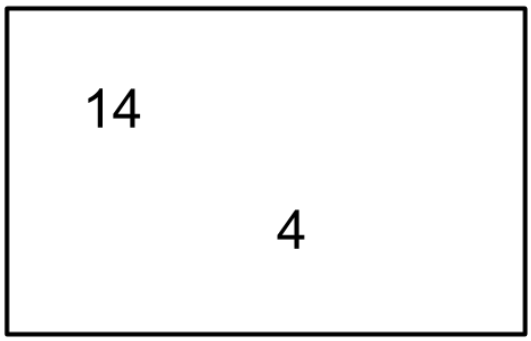
7. On the pool table shown below, a pool ball must be shot initially from the point $(0, 0)$ and must roll along the side to either $(0, 3)$ or $(5, 0)$. Each time the ball strikes another side, it will bounce off at an angle of $60°$ as indicated by the dotted lines. First, start at $(0, 0)$, and shoot the ball towards $(5, 0)$, and record the coordinates each time the ball strikes a side. Then start at $(0, 0)$, shoot the ball towards $(0, 3)$, and record the coordinates again. Can you relate your results to the previous problem?



(0,0)	(0,0)
(5,0)	(0,3)
(2,3)	(,)
(,)	(,)
(,)	(,)
(,)	(,)
(,)	(,)
	(,)
	(,)

Euclid's Game

Start with two numbers in a box. Two players take turns writing a new number in the box that is the difference of two existing numbers in the box and is a positive number (not zero or a negative number). The player who can no longer make a move loses.



1. Is there a winning strategy for Euclid's Game?
2. Based on the two starting numbers, how can you predict if the first player or second player will win, assuming optimal play?

Additional Problems

1. Josie has 32 cookies and 48 pretzels to put in party favor bags at her birthday party. If she needs to put the same number of cookies in each bag and the same number of pretzels in each bag, what is the largest number of bags she can use (so that she can invite as many friends as possible)?
2. Abe is going to plant 63 apple trees and 27 peach trees in rows that all have the same number of trees and are made up of only one type of tree. What is the greatest number of trees that Abe can have in each row?
3. What is the greatest common divisor of
   (a) 48 and 52?
   (b) 2000 and 7200?
   (c) 847 and 539?
   (d) 221 and 323?