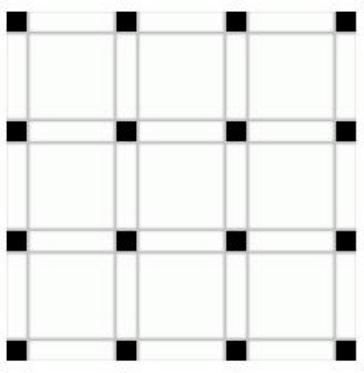
Math Circle Worksheet
Dots and Boxes, pt. 1

11/17/18

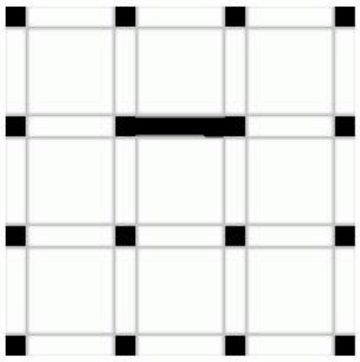
1 Introduction

Dots-and-boxes is a two player game played on a rectangular grid of dots. Players A and B take turns drawing an edge between two adjacent dots (dots are adjacent if and only if they are above/below one another, or to the left/right of one another; not diagonally across). When four edges are drawn around a square, and that square does not contain any dots in its interior, then the player that drew the last line gets to write their initial in said square. The game continues until all possible edges have been drawn, and the player with the most boxes "claimed" wins.

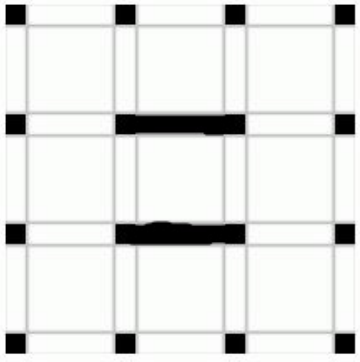
Here is an example. In this worksheet we'll refer to a grid by the number of rows and columns of boxes it could possibly have, not by the number of dots along the edges. The following grid is a 3×3 grid:



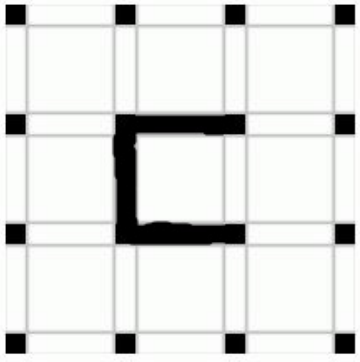
In dots-and-boxes, A takes the first move; it might look like this:



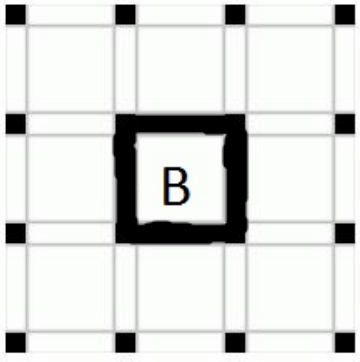
Then B may draw an edge like so:



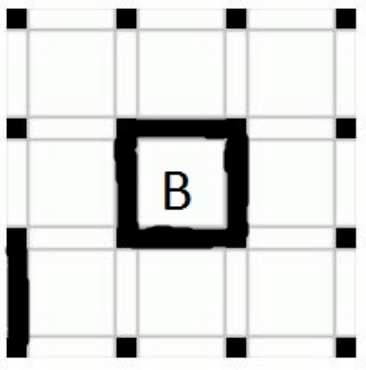
A draws another nearby edge:



And finally, B draws the last edge to fill in a box:



Since B was able to complete a square, B can go again:



Etc.

In this worksheet, we'll take a step towards understanding, mathematically, what is happening in this simple game. The first part of this session consists of you playing the game with a neighbor. A few rectangular dotted grids have been included with these worksheets, together with some questions; as you play against your peers, keep these questions in mind.

2 Questions to think about:

- Is there a winning strategy for the 2×2 board, for the first or second player? If so, what is it?
- Is there a winning strategy for the 2×3 board, for either player? If so, what is it?
- Can you think of any general winning strategies?
- As you play against others, what kinds of configurations appear in the grid?