# Geometry for Middle School Math Competitions
- ### Chapel Hill Math Circle

In this warm-up activity, we will explore a few of the most famous and most frequently used geometry patterns that you can use in math competitions. Then we'll try some MathCounts Geometry problems.

1. Use the Pythagorean Theorem to find the length of the diagonal of a square with side length of 10. Express your answer in simplest radical form. **(Not sure about 'Pythagorean Theorem" or 'Simplest Radical Form"? - ask the person beside you or a volunteer.)** Repeat the process for a square with side lengths of 7 units.

2. Use the Pythagorean Theorem to find the height of an equilateral triangle with side lengths of 10. Express your answer in simplest radical form. Repeat the process if the side length was 7.

3. The most common and famous "Pythagorean Triple" is the 3-4-5 right triangle. These three sides make a right triangle because $3^2 + 4^2 = 5^2$ $(9 + 16 = 25)$. Most right triangles have at least one side that is an irrational number such as the 5-6-$\sqrt{61}$ right triangle. We call it a Pythagorean Triple when all three numbers are nice, clean natural numbers. List other Pythagorean Triples that you know of or can discover.

4. The three interior angles of a triangle (n=3) sum to 180°. The four interior angles of a quadrilateral (n=4) sum to 360°. If this is the beginning of a pattern, what would the sum be for pentagons (n=5) or hexagons (n=6)? Can you state the pattern as a rule in terms of n?

5. If the polygon is "regular" (equilateral and equiangular), we can turn the rule above into a rule for the measure of each angle of a regular polygon with n sides. What is that rule? What would be the measure of each interior angle of a regular pentagon? A regular hexagon?

6. Regular hexagons are the most famous regular polygons because the diagonals split the inside into 6 identical equilateral triangles. Sketch a regular hexagon below, including its diagonals. What would be the area of this polygon if the side length were 10 units?
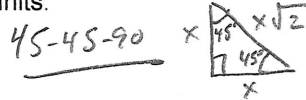
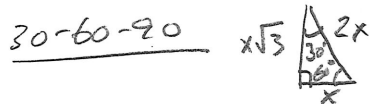**Geometry for MIddle School Math Competitions**
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    45-45-90

2.  Use the Pythagorean Theorem to find the height of an equilateral triangle with side lengths of 10. Express your answer in simplest radical form. Repeat the process if the side length was 7.

    30-60-90

3.  The most common and famous "Pythagorean Triple" is the 3-4-5 right triangle. These three sides make a right triangle because $3^2 + 4^2 = 5^2$ $(9 + 16 = 25)$. Most right triangles have at least one side that is an irrational number such as the 5-6-$\sqrt{61}$ right triangle. We call it a Pythagorean Triple when all three numbers are nice, clean natural numbers. List other Pythagorean Triples that you know of or can discover.

    3-4-5    5-12-13    7-24-25    8-15-17    9-40-41    12-35-37
    6-8-10   10-24-26

4.  The three interior angles of a triangle (n=3) sum to 180°. The four interior angles of a quadrilateral (n=4) sum to 360°. If this is the beginning of a pattern, what would the sum be for pentagons (n=5) or hexagons (n=6)? Can you state the pattern as a rule in terms of n?

    $(n-2) \cdot 180$

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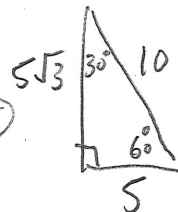
    $$\frac{(n-2) \cdot 180}{n}$$

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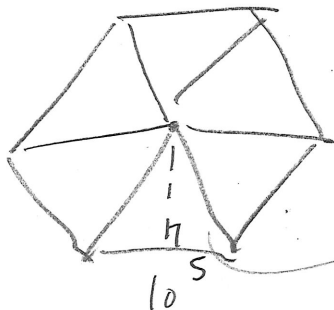
    Also, Area of equilateral △

    $$A = \frac{x^2\sqrt{3}}{4}$$

    $$A = \frac{5(5\sqrt{3})}{2} = \frac{25\sqrt{3}}{2}$$

    $$A = 12\left(\frac{25\sqrt{3}}{2}\right) = 150\sqrt{3}$$