2007-08 Math Counts - Coordinate Plane Name _____

LEAP Math w/ Dr. Blackburn

u6 10. _____ units Circles with centers at (2, 2) and (17, 10) are both tangent to the x-axis. What is the distance between the closest points of the two circles?

u6 7. _____ sq units What is the area of the quadrilateral with vertices at (1, 1), (5, 2), (4, 4) and (2, 3)?

u9 10. (_____, _____) The graphs of $y = 2x^3$ and $y = 3x^2$ intersect at (0, 0) and at what other point? Express your answer as an ordered pair of common-fraction coordinates.

u-11 7. _____ The point (8, 10) is the same distance from the point (0, y) as it is from the x-axis. What is the greatest possible value of y?

u-12 7. _____ The function $f(x) = 3x^2 - 6x - 11$ is graphed on a coordinate plane. What is the smallest y-coordinate of any points of the function?

u-13 4. _____ sq units What is the area of the region enclosed by the graphs of the lines $y = -2x - 3$, $y = 2x - 3$ and the x-axis? Express your answer as a decimal to the nearest tenth.

u-15 5. (_____, _____) The point A($\frac{5}{2}$, 0) is reflected over the line $y = \frac{1}{2}x$ to the point A'. What are the coordinates of A'? Express any non-integer coordinate as a common fraction.

u-14 3. _____ If the graph of $f(x) = 2x^2 + bx - 3$ is symmetric about the line $x = 3$, what is the value of b?

u-14 7. _____ sq units What is the area of a triangle with vertices at (-5, -1), (3, 5) and (1, 9) on the coordinate plane?

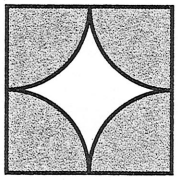
u-14 5. _____ points Point A is located at (5, 5) on the Cartesian plane. Point B is also in the plane and has integer coordinates. If $0 < AB \leq 4$, at how many points could B be located?

2007-08 Math Counts Geometry (1)

LEAP Math w/ Dr. Blackburn

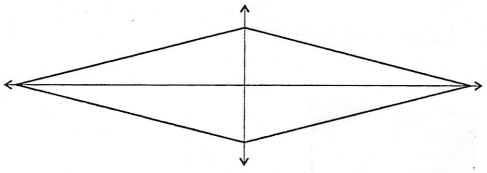
Name _____

U4 2. _____ sq cm Four congruent quarter-circles are drawn inside a square of side length 4 centimeters, as shown. What is the area of the shaded portion of the square region? Express your answer in terms of π.



U8 2. _____ sq units What is the area of the triangle with vertices at (0, 2), (3, 2) and (3, 0) on the coordinate plane?

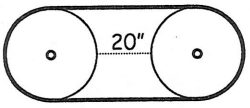
U10 7. _____ sq units



What is the area enclosed on the coordinate plane by the graph of the equation $|x| + |4y| = 20$?

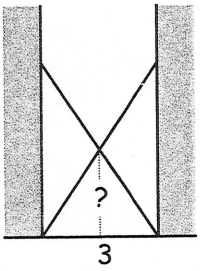
U10 2. _____ cu in A rectangular block has faces with areas of 48 in^2, 72 in^2 and 96 in^2. What is the volume of the block?

O4 8. _____ inches



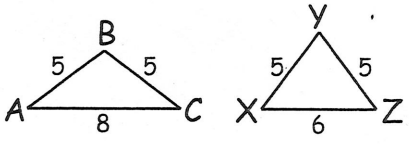
Each circular pulley shown has a radius of 12 inches. The shortest distance between the pulleys is 20 inches. What is the length of the continuous belt that loops around both pulleys? Express your answer to the nearest whole number.

O4 9. _____ meters Two ladders, both 6 meters in length, are leaned up against opposite vertical walls in a 3-meter-wide corridor, as shown. How far above the ground do the two ladders cross? Express your answer as a decimal to the nearest tenth.

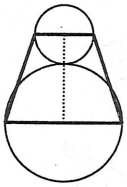


U-4 9. _____ Several points are plotted on a graph. For each point, the x-coordinate is the length of a side of a square while the y-coordinate is the perimeter of that same square. One such point is (2, 8) since a square with side length 2 units has a perimeter of 8 units. What is the slope of the line connecting the points? Express your answer in simplest form.

U-4 10. _____ sq units Triangle ABC has side lengths 5, 5 and 8 units. Triangle XYZ has side lengths 5, 5 and 6 units. What is the difference of the areas of these two triangles?



U9 4. _____ sq units



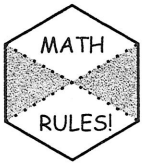
Two non-congruent circles are externally tangent to each other. Each base of an isosceles trapezoid is a diameter of one of the circles. If the distance between the centers of the circles is 9 units, what is the area of the trapezoid?

2007-08 Math Counts Geometry (2)

LEAP Math w/ Dr. Blackburn

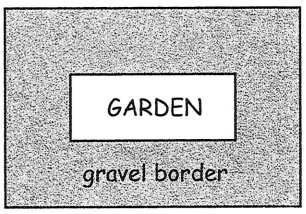
Name _____

u8 7. _____ sq inches The area of this sign in the shape of a regular hexagon is $96\sqrt{3}$ square inches. What is the total area of the two shaded regions? Express your answer in simplest radical form.

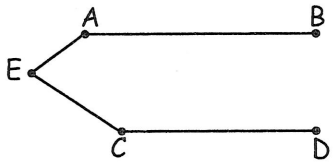


u8 8. _____ sq units Two sides of a particular isosceles triangle are 6 and 13 units. What is the area of this triangle? Express your answer in simplest radical form.

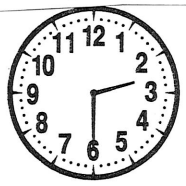
u8 9. _____ feet A rectangular garden is 10 feet by 4 feet. A gravel border with uniform width along the sides and 90° corners surrounds the garden, as shown. The area of the gravel border is six times the area of the garden. What is the perimeter of the outside of the gravel border?



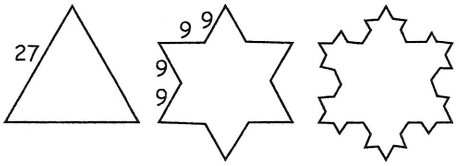
u8 10. _____ degrees In the figure shown, segments AB and CD are parallel. What is the sum of the measures of angles BAE, AEC and ECD?



u9 2. _____ degrees How many degrees are there in the smallest angle between the two hands of a clock at 2:30?



u7 7. _____ units In the sequence of equilateral figures shown, the middle third of each segment is replaced with two segments that are each the same length as the replaced piece. Each side of the first figure (the triangle) is 27 units. What is the perimeter of the third figure in the sequence?



u7 3. _____ sq units What is the area of regular hexagon ABCDEF if AB = 2 units? Express your answer in simplest radical form.

05 8. _____ sq ft A 20-foot-high rectangular room has a floor that measures 18' by 15'. Its doorway measures 3' by 12', and its only window measures 7' by 10'. How many square feet of wall space does the room have?

05 2. _____ sq units An acute angle of a right triangle is 30°, and the hypotenuse is 40 units. What is the area of the triangle? Express your answer as a decimal to the nearest tenth.

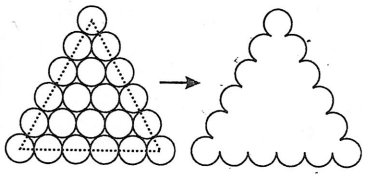
03 1. _____ feet The formula for the total surface area of a cylinder is $SA = 2\pi r^2 + 2\pi rh$, where r is the radius and h is the height. A particular solid right cylinder of radius 2 feet has a total surface area of 12π square feet. What is the height of this cylinder?

2007-08 Math Counts - Geometry 3

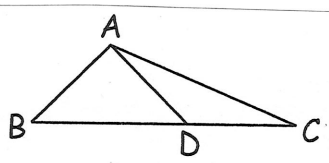
Name _____

LEAP Math w/ Dr. Blackburn

02 10. _____ cm Twenty-one congruent circular discs are stacked in a triangular arrangement, as shown. Connecting the centers of the three vertex discs forms an equilateral triangle. The circumference of each disc is 18 cm. What is the outside perimeter of the arrangement?



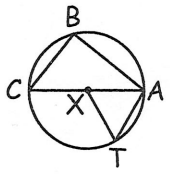
02 8. _____ degrees



In the figure, BA = AD = DC and point D is on segment BC. The measure of angle ADC is 135 degrees. What is the measure of angle ABC?

03 10. _____ sq in In triangle ABC the length of the altitude to side AB is 6 inches, and the measures of angles A and B are 45 degrees and 60 degrees, respectively. What is the area of triangle ABC? Express your answer as a decimal to the nearest tenth.

US 2. _____ units In the circle with center X, the measure of angle AXT is 60°, and the measure of angle ABC is 90°. The length of segment AT is 5 units, and the length of segment BC is 6 units. What is the length of segment AB?



US 3. _____ sq ft A 40-foot by 10-foot rectangular garden is enclosed by a fence. To make the garden larger, while using the same amount of fencing, its shape is changed to a square. How many square feet larger than the old garden is the new garden?

03 2. _____ A cube varies in size over time, with its largest volume being twice its smallest volume. What is its largest edge length divided by its shortest edge length? Express your answer as a decimal to the nearest hundredth.

02 3. _____ watts Lighting experts recommend 150 to 200 watts of illumination for every 50 square feet of floor space. What is the minimum number of watts recommended for a room with a rectangular floor measuring 30 feet by 40 feet?

US 10. _____ units A triangle has sides of length 5 and 6 units. The length of the third side is x units, where x is an integer. What is the largest possible perimeter of the triangle?

0-64. _____ feet A farmer's field is in the shape of regular hexagon ABCDEF. The distance from point A to point B is 420 feet. A fence post is placed at each vertex of the hexagon, and each side has 16 evenly spaced fence posts (counting the posts at the vertices). What is the distance from the center of one fence post to the center of an adjacent fence post on a side of the hexagonal field?

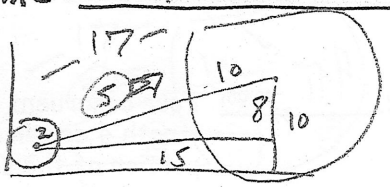
0-65. _____ sq cm A circle with a radius of 2.5 cm is inscribed in a square. What is the area within the square region but outside the circular region? Express your answer as a decimal to the nearest tenth.

2007-08 Math Counts - Coordinate Plane

checked ✓

Name Boyd's Work.

LEAP Math w/ Dr. Blackburn

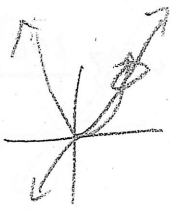


U6 10. 5 ✓ units Circles with centers at (2, 2) and (17, 10) are both tangent to the x-axis. What is the distance between the closest points of the two circles?

U6 7. 6 ✓ sq units What is the area of the quadrilateral with vertices at (1, 1), (5, 2), (4, 4) and (2, 3)?

(2 - 1 - 2 - 1 - 1 - 1

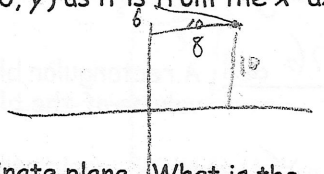
U9 10. $\frac{3}{2}, \frac{27}{4}$ ✓ The graphs of $y = 2x^3$ and $y = 3x^2$ intersect at (0, 0) and at what other point? Express your answer as an ordered pair of common-fraction coordinates.



$2x^3 = 3x^2$ $2x = 3$ $x = \frac{3}{2}$ $y = 3\left(\frac{9}{4}\right)$ $y = \frac{27}{4}$

16 ✓

U-11 7. 14 ✗ The point (8, 10) is the same distance from the point (0, y) as it is from the x-axis. What is the greatest possible value of y?

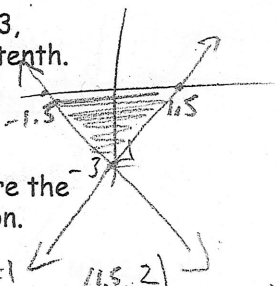


U-12 7. -14 ✓ The function $f(x) = 3x^2 - 6x - 11$ is graphed on a coordinate plane. What is the smallest y-coordinate of any points of the function?

when $x = 1$, $y = (3 - 6 - 11)$ $y = -14$

$\frac{-b}{2a} = \frac{6}{6} = 1$

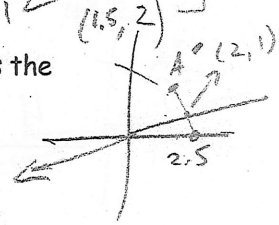
U-13 4. 4.5 ✓ sq units What is the area of the region enclosed by the graphs of the lines $y = -2x - 3$, $y = 2x - 3$ and the x-axis? Express your answer as a decimal to the nearest tenth.



U-15 5. $\frac{3}{2}, 2$ ✓ The point $A(\frac{5}{2}, 0)$ is reflected over the line $y = \frac{1}{2}x$ to the point A'. What are the coordinates of A'? Express any non-integer coordinate as a common fraction.

2.5

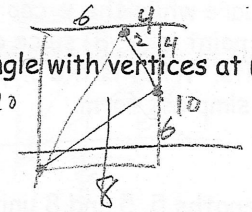
$y - 0 = -2\left(x - \frac{5}{2}\right)$ $y = -2x + 5$ $\frac{1}{2}x = -2x + 5$ $2.5x = 5$ $x = 2, y = 1$



U-14 3. -12 ✓ If the graph of $f(x) = 2x^2 + bx - 3$ is symmetric about the line $x = 3$, what is the value of b?

$\frac{-b}{2a} = 3$ $\frac{-b}{4} = 3$ $-b = 12$ $b = -12$

U-14 7. 22 ✓ sq units What is the area of a triangle with vertices at (-5, -1), (3, 5) and (1, 9) on the coordinate plane?

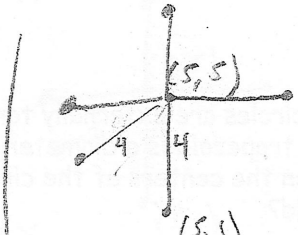


$m = \frac{6}{4}$ $m = \frac{4}{2}$ not 1

80 - 30 - 24 - 4

48 ✗

U-14 5. 44 points Point A is located at (5, 5) on the Cartesian plane. Point B is also in the plane and has integer coordinates. If $0 < AB \leq 4$, at how many points could B be located?

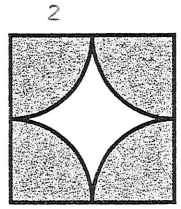


(congruent pieces)

2007-08 Math Counts Geometry (1) Name Boyd's Work

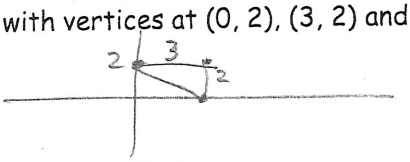
LEAP Math w/ Dr. Blackburn

u4 2. 4π sq cm ✓ Four congruent quarter-circles are drawn inside a square of side length 4 centimeters, as shown. What is the area of the shaded portion of the square region? Express your answer in terms of π.

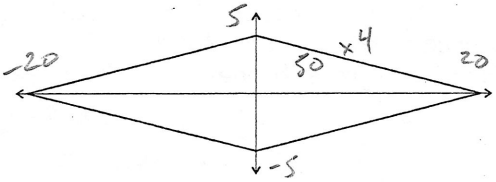


4π

u8 2. 3 sq units ✓ What is the area of the triangle with vertices at (0, 2), (3, 2) and (3, 0) on the coordinate plane?



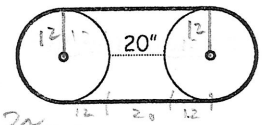
u10 7. 200 sq units ✓



What is the area enclosed on the coordinate plane by the graph of the equation $|x| + |4y| = 20$?

u10 2. 576 cu in ✓ A rectangular block has faces with areas of 48 in², 72 in² and 96 in². What is the volume of the block? 16×3 8×6 6×12 8×12 $6\times8\times12$

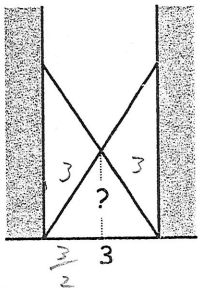
04 8. 163 inches ✓



Each circular pulley shown has a radius of 12 inches. The shortest distance between the pulleys is 20 inches. What is the length of the continuous belt that loops around both pulleys? Express your answer to the nearest whole number.

$2\pi r + 2(44)$ $24\pi + 88 = 163.4$

04 9. 2.6 meters ✓ Two ladders, both 6 meters in length, are leaned up against opposite vertical walls in a 3-meter-wide corridor, as shown. How far above the ground do the two ladders cross? Express your answer as a decimal to the nearest tenth.

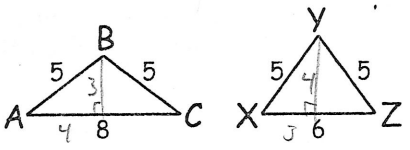


$\frac{3}{2}\sqrt{3} = 2.6$

u-4 9. 4 ✓ Several points are plotted on a graph. For each point, the x-coordinate is the length of a side of a square while the y-coordinate is the perimeter of that same square. One such point is (2, 8) since a square with side length 2 units has a perimeter of 8 units. What is the slope of the line connecting the points? Express your answer in simplest form.

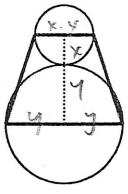
1, 4
2, 8
3, 12
$m = 4$

u-4 10. 0 sq units ✓ Triangle ABC has side lengths 5, 5 and 8 units. Triangle XYZ has side lengths 5, 5 and 6 units. What is the difference of the areas of these two triangles?



12 12

u9 4. 81 sq units ✓



Two non-congruent circles are externally tangent to each other. Each base of an isosceles trapezoid is a diameter of one of the circles. If the distance between the centers of the circles is 9 units, what is the area of the trapezoid?

$x + y = 9$ $A = \left(\frac{b_1 + b_2}{2}\right) \cdot 9$ $A = 9 \cdot 9$ $= 81$

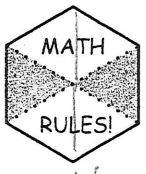
2007-08 Math Counts Geometry (2)

LEAP Math w/ Dr. Blackburn

checked 10/10 Name Boyd's Work

u8 7. $32\sqrt{3}$ sq inches ✓

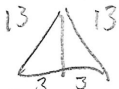
The area of this sign in the shape of a regular hexagon is $96\sqrt{3}$ square inches. What is the total area of the two shaded regions? Express your answer in simplest radical form.



$16\sqrt{3} \cdot 2$

u8 8. $12\sqrt{10}$ sq units ✓

Two sides of a particular isosceles triangle are 6 and 13 units. What is the area of this triangle? Express your answer in simplest radical form.

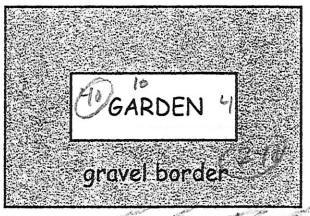


$h = \sqrt{160}$, $h = 4\sqrt{10}$

$\frac{6h}{2}$ $\frac{6 \cdot 4\sqrt{10}}{2}$

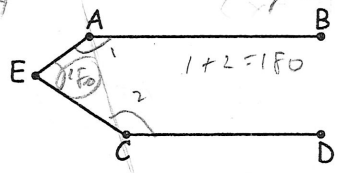
u8 9. 68 feet ✓

A rectangular garden is 10 feet by 4 feet. A gravel border with uniform width along the sides and 90° corners surrounds the garden, as shown. The area of the gravel border is six times the area of the garden. What is the perimeter of the outside of the gravel border?



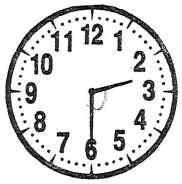
u8 10. 360 degrees ✓

In the figure shown, segments AB and CD are parallel. What is the sum of the measures of angles BAE, AEC and ECD?



u9 2. 105 degrees ✓

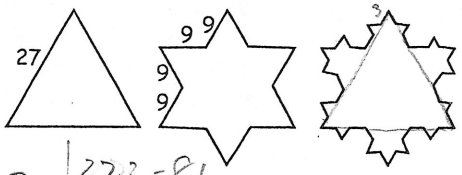
How many degrees are there in the smallest angle between the two hands of a clock at 2:30?



$30° \cdot 3 = 90 + 15 = 105$

u7 7. 144 units ✓

In the sequence of equilateral figures shown, the middle third of each segment is replaced with two segments that are each the same length as the replaced piece. Each side of the first figure (the triangle) is 27 units. What is the perimeter of the third figure in the sequence?

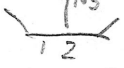


0	$27 \cdot 3 = 81$
1	$36 \cdot 3 = 108$
2	$48 \cdot 3 = 144$

$16 \cdot 3 = 48$

u7 3. $6\sqrt{3}$ sq units ✓

What is the area of regular hexagon ABCDEF if AB = 2 units? Express your answer in simplest radical form.



$A = \frac{\sqrt{3} \cdot 12}{2} = 6\sqrt{3}$

05 8. 1214 sq ft ✓

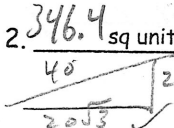
A 20-foot-high rectangular room has a floor that measures 18' by 15'. Its doorway measures 3' by 12', and its only window measures 7' by 10'. How many square feet of wall space does the room have?

$15 \times 20 (\times 2) + 18 \times 20 (\times 2) = 1320 - 36 - 70$

$600 + 720$

05 2. 346.4 sq units ✓

An acute angle of a right triangle is 30°, and the hypotenuse is 40 units. What is the area of the triangle? Express your answer as a decimal to the nearest tenth.



$200\sqrt{3} = 346.4$

03 1. 1 feet

The formula for the total surface area of a cylinder is $SA = 2\pi r^2 + 2\pi rh$, where r is the radius and h is the height. A particular solid right cylinder of radius 2 feet has a total surface area of 12π square feet. What is the height of this cylinder?

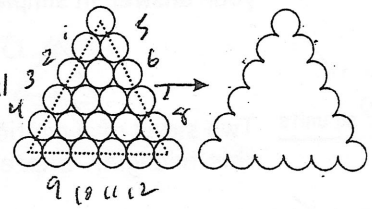
$12\pi = 2\pi \cdot 4 + 4\pi h$ $4\pi = 4\pi h$ $h = 1$

$12\pi = 8\pi + 4\pi h$

2007-08 Math Counts - Geometry 3 (checked 9/10) Name Boyd's Work

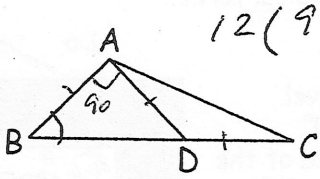
LEAP Math w/ Dr. Blackburn

02 10. 153 cm ✓ Twenty-one congruent circular discs are stacked in a triangular arrangement, as shown. Connecting the centers of the three vertex discs forms an equilateral triangle. The circumference of each disc is 18 cm. What is the outside perimeter of the arrangement?



$12(9) + 3\left(\frac{5}{6}\right)(18)$

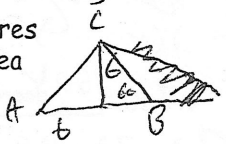
02 8. ~~112.5~~ degrees 45 ✓ In the figure, BA = AD = DC and point D is on segment BC. The measure of angle ADC is 135 degrees. What is the measure of angle ABC?



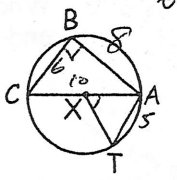
$180 - 135 = \frac{45}{2} = 22.5$

90 + 22.5 = 112.5

03 10. 28.4 sq in ✓ In triangle ABC the length of the altitude to side AB is 6 inches, and the measures of angles A and B are 45 degrees and 60 degrees, respectively. What is the area of triangle ABC? Express your answer as a decimal to the nearest tenth.



05 2. 8 units ✓ In the circle with center X, the measure of angle AXT is 60°, and the measure of angle ABC is 90°. The length of segment AT is 5 units, and the length of segment BC is 6 units. What is the length of segment AB?



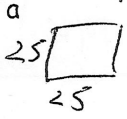
$x\sqrt{3} = 6$

$x = \frac{6}{\sqrt{3}}$

$x = 3.464$

05 3. 225 sq ft ✓ A 40-foot by 10-foot rectangular garden is enclosed by a fence. To make the garden larger, while using the same amount of fencing, its shape is changed to a square. How many square feet larger than the old garden is the new garden?

100



625 − 400 = 225

03 2. 1.26 ✓ A cube varies in size over time, with its largest volume being twice its smallest volume. What is its largest edge length divided by its shortest edge length? Express your answer as a decimal to the nearest hundredth.

$\sqrt[3]{100}$ / $\sqrt[3]{50}$

02 3. 3600 watts ✓ Lighting experts recommend 150 to 200 watts of illumination for every 50 square feet of floor space. What is the minimum number of watts recommended for a room with a rectangular floor measuring 30 feet by 40 feet?

1200

$150 \times 24 =$

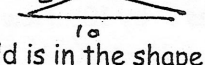
24

50 | 1200

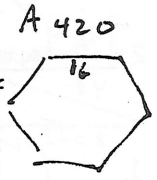
100

200

05 10. 21 units ✓ A triangle has sides of length 5 and 6 units. The length of the third side is x units, where x is an integer. What is the largest possible perimeter of the triangle?



0-64. 28 feet ✓ A farmer's field is in the shape of regular hexagon ABCDEF. The distance from point A to point B is 420 feet. A fence post is placed at each vertex of the hexagon, and each side has 16 evenly spaced fence posts (counting the posts at the vertices). What is the distance from the center of one fence post to the center of an adjacent fence post on a side of the hexagonal field?



$\frac{420}{15} = 28$

0-65. 5.4 sq cm ✓ A circle with a radius of 2.5 cm is inscribed in a square. What is the area within the square region but outside the circular region? Express your answer as a decimal to the nearest tenth.

$25 - \pi(2.5)^2$