Math Circle Worksheet
The Game of Hex, pt. 1

10/20/18

1 Introduction

This two part worksheet will introduce you to the game of Hex. In the first part, you will play Hex with other Math Circlers to get a feel for the game, and get a sense of strategies that can be used to win. In the second part, we will explore more in-depth the question of whether or not Hex can end in a tie.

2 The game of Hex

In the version of Hex we will look at today, each board will be a $k \times k$ hexagonal (made from hexagons) grid, where k will either be $3, 4, 6$, or 11. Two opposite edges of the grid are marked with an 'X' (or red), and the other two edges will be marked with an 'O' (or black). Two players take turns marking hexagons either 'X' or 'O' until there is a path of the same symbol (or same colour) connecting opposite edges, where two hexagons are adjacent if they share an edge. For example, the 'X' player will win if there is a chain of hexagons marked 'X' connecting the two 'X' edges.

Take some time playing Hex with a peer; a number of empty Hex boards are provided on another handout. While you're playing Hex, keep in mind the following questions:

- Is there a winning strategy for the 3×3 board? What is it?
- Is there a winning strategy for the 4×4 and 6×6 board? What are they?
- What are strategies you came up with when playing Hex with others?
- How do these strategies change as the board size increases?
- Is a tie possible?