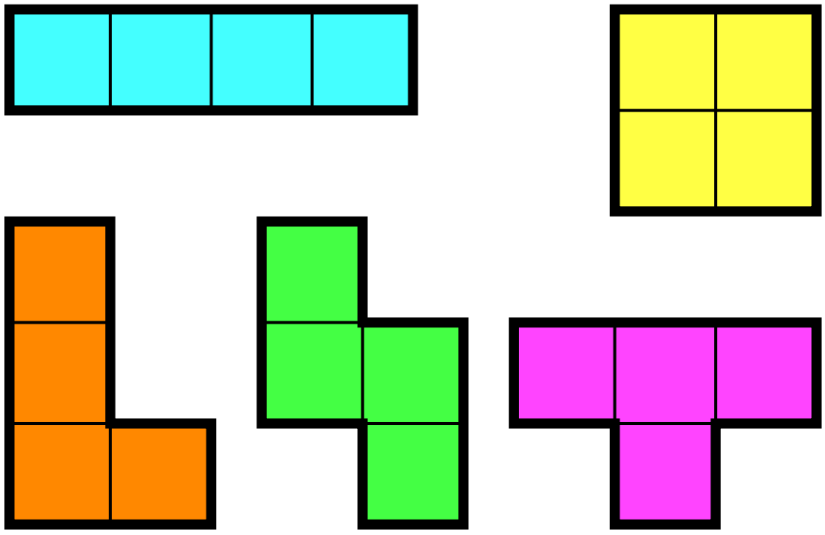
Knights and Knaves

September 22, 2018

From last time: Tiling with Tetrominoes

Last time, we found that there are 5 tetrominoes.



For each of these cases, show a way to cover the board or explain why it cannot be done.

1. Can you tile a 20×1 board with all the tetrominoes, using each one once?

2. A 10x2?

3. A 4x5?

And, Or, and Not

1. Your friend says she will bring apple slices and orange wedges to school the next day to share. Did she keep her promise if she:
   - Brought only apple slices?
   - Brough only orange wedges?
   - Brought only bananas?
   - Brought apple slices and orange wedges?
2. Your teacher says, "you must do some reading or do some math"
   - Do you need to do both?
   - Is it ok if you do both?
   - Is it ok if you do neither?
3. Your friend says, "I'm going to marry Harry or Joe."
   - Do you think she will let you have both?
4. Your mom says, "You can have ice cream or cake for dessert."
   - Do you think she will let you have both?
5. Linda: I either have a dollar in my pocket or a frog in my hand.
   Mary: That's not true!

   If Mary is correct, then what do you know about what is in Linda's pocket and what is in her hand?
6. Linda: I have at least one flavor of ice cream in my freezer.
   Mary: That's not true!

   If Mary is correct again, then what do you know about Linda's freezer?
7. Linda: I have at least three kinds of vegetables in my refrigerator.
   Mary: That's not true!

   If Mary is correct again, then what do you know about Linda's freezer?

Knights and Knaves - Two residents [1]

On the Island of Knights and Knaves, there are two kinds of inhabitants: knights, who always tell the truth, and knaves, who always lie.

8. Would a person from the island of Knights and Knaves ever say "I am a Knave"?

In the following problems, A and B are residents of the Island of Knights and Knaves.

9. A makes the following statement: "At least one of us is a knave."
   What are A and B?
10. Suppose A says, "Either I am a knave or B is a knight." What are A and B?
11. Suppose A says, "Either I am a knave or else two plus two equals five." What would you conclude?
12. Suppose A says, "I am a knave, but B isn't." What are A and B?
13. Once Ms. Mary visited the island of knights and knaves, and came across two of the inhabitants resting under a tree. She asked one of them, "Is either of you a knight?" He answered, and Ms. Mary knew the answer to her question.
    What is the person to whom Ms. Mary addressed the question – is he a knight or a knave? And what is the other one? You have enough information to answer the question.
14. Suppose *you* visit the island of knights and knaves. You come across two of the inhabitants lazily lying in the sun. You ask one of them whether the other one is a knight, and you get a yes-or-no answer .Then you ask the second one whether the first one is a knight. You get a yes-or-no answer. Are the two answers necessarily the same?

Knights and Knaves, Three Residents

In the following problems, A, B, and C are all residents of the Island of Knights and Knaves.

15. Three of the inhabitants of the island of knights and knaves – A, B, and C – were standing together in a garden. A stranger passed by and asked A, "Are you a knight or a knave?" A answered, but rather indistinctly, so the stranger could not make out what he said. The stranger then asked B, "What did A say?" B replied, "A said that he is a knave." At this point the third man, C, said "Don't believe B; he is lying!"
    The question is, what are B and C?
16. Suppose the stranger, instead of asking A what he is, asked A. "How many knights are among you?" Again, A answers indistinctly. So the stranger asks B, "What did A say?" B replies, "A said that there is one knight among us." Then C says, "Don't believe B; he is lying!"
    Now what are B and C?

[1] These problems are all taken from *What Is the Name of This Book* by Raymond Smullyan

17. Again we have three people A, B, C, each of whom is either a knight or a knave. A and B make the following statements:

    A: All of us are knaves.
    B: Exactly one of us is a knight.

    What are A, B, and C?

18. Suppose instead, A and B say the following:

    A: All of us are knaves.
    B: Exactly one of us is a knave.

    Can it be determined what B is? Can it be determined what C is?

19. We again have three inhabitants, A, B, and C, each of whom is a knight or a knave. Two people are said to be of the *same type* if they are both knights or both knaves. A and B make the following statements:

    A: B is a knave.
    B: A and C are of the same type.

    What is C?

20. Again three people A, B, and C. A says "B and C are of the same type." Someone then asks C, "Are A and B of the same type?" What does C answer?

Extra Problems: Knights, Knaves, and Normals

On another island, there are knights, knaves, and normals. The knights always tell the truth, the knaves always lie, and the normals sometimes tell the truth and sometimes lie.

21. We are given three people, A, B, C, one of whom is a knight, one a knave, and one normal (but not necessarily in that order). They make the following statements.

    A: I am normal.
    B: That is true.
    C: I am not normal.

    What are A, B, and C?

22. Two people, A and B, each of whom is either a knight, or knave, or a normal, make the following statements:

    A: B is a knight.
    B: A is not a knight.

    Prove that at least one of them is telling the truth, but is not a knight.

23. This time A and B say the following:

    A: B is a knight.
    B: A is a knave.

    Prove that either one of them is telling the truth but is not a knight, or one of them is lying but is not a knave.

24. On this island of knights, knaves, and normals, knaves are said to be of the *lowest* rank, normals of the *middle* rank, and knights of the *highest* rank.

    Given two people A, B, each of whom is a knight, a knave, or a normal, they make the following statements.

    A: I am of lower rank than B.
    B: That's not true!

    Can the ranks of either A or B be determined? Can it be determined, of either of these statements, whether it is true or false?

25. Given three people A, B, C, one of whom is a knight, one a knave, and one normal. A, B make the following statements:

    A: B is of higher rank than C. B: C is of higher rank than A.

    Then C is asked: "Who has higher rank, A or B?" What does C answer?