Math Auction

1   Rules of the Math Auction

1. We divide into teams and work for a fixed amount of time to solve the problems below.

2. Each team is given $200 to start.

3. The best solution to a problem is worth $100.

4. The problems are put up for auction in the order given. The team with the highest bid is allowed to present its solution.

5. The problem is then put up for bid again (and again), but each time the solution must be better than the previous solution.

6. When no other team wants to buy the problem, the team with the best solution collects the value of the problem. Every team that bought the problem pays for its bid, even if it did not have the winning solution.

2 Problems

1. Find as many different ways as possible to cut this figure into four pieces of the same size and shape, using straight line segments that begin and end at grid points.
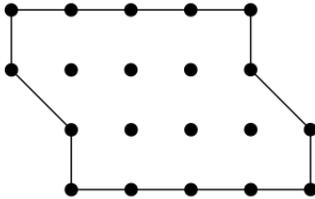
2. Suppose we have a balance (i.e. a scale with two arms, which can only tell us whether the items place on one arm weigh less than or more than the items place on the other arm). We are given 12 coins that look identical, but one coin is counterfeit and does not have the same weight as the real coins. We do not know if the counterfeit coin is heavier or lighter than the real coins. Present an algorithm to find the counterfeit coin AND whether it is lighter or heavier in the fewest number of weighings as possible in the worst case.
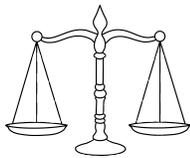
3. Six children sit around a circular table with 6 seats. Find as many different ways as possible to rearrange the children so that no child moves more than one seat to the right or to the left of his or her original position.

4. Make the minimum possible number of marks on a wooden plank so that every integer number of inches from 1 through 15 can be measured using this plank; that is, this length can be represented as the distance between some pair of the marks, or between one end of the plank and a mark.

5. Ali the trader has a heavy gold bracelet with 11 links in it. During one of his trips, he decides to stay at a local inn for 11 days. The innkeeper asks him for one gold link per day as payment. The trader does not want to pay more that he owes, while the innkeeper wants to get his payment daily. However, the innkeeper is willing to trade the links to make even. For example, if Ali pays with the a single link on day one, he can give a chain with two links on day two and get the single link back. The trader will have to cut his bracelet into several pieces by breaking a few of its links. What is the minimum number of links Ali needs to cut?