Two is the Best Number (Part 3)

1 Warm-Up

1. In this problem there is a long straight road.

   (a) A cricket jumps along the road. Every jump takes him 1 meter to the right or to the left. Could it be that in 21 jumps he will end up exactly where he started? Explain.

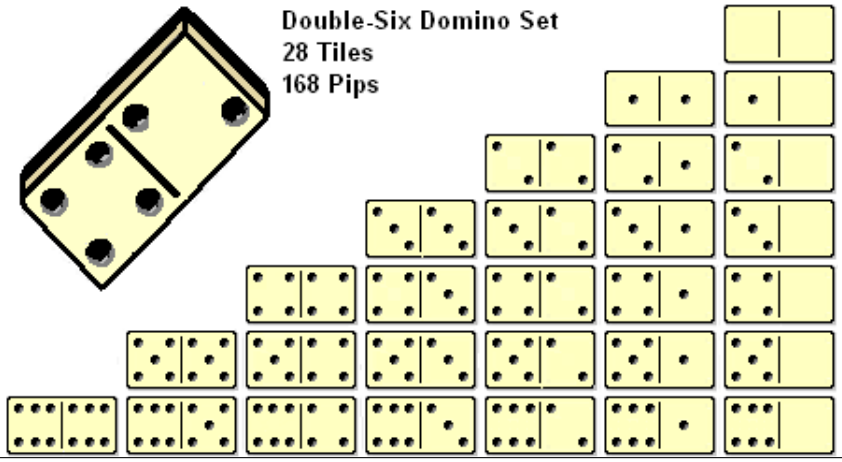
   (b) A brown cricket and a green cricket are sitting on the road 9 meters apart. They start jumping at exactly the same moment. On every jump, each of them leaps 1 meter to the right or to the left. Whenever one jumps, the other jumps too, at the same time. Can they ever land on the same spot simultaneously? Explain.

   (c) A brown cricket and a green cricket are sitting on the road 15 meters apart. They start jumping at exactly the same moment. On every jump, each of them leaps 3 meters to the right or to the left. Whenever one jumps, the other jumps too, at the same time. Can they ever land on the same spot simultaneously? Explain.

2 Dominos

2. Kolya placed a full set of dominos (all 28 pieces) into a line according to the rule of dominos (see below). The left half of the leftmost domino has 6 spots on it. How many spots are there on the right half of the rightmost domino?

   Rule of Dominos: When two dominos are placed next to each other, end to end, the halves that are next to each other must have the same number of spots.



This week's problems are from *Mathematical Circle Diaries, Year 1* by Anna Burago, from *Mathematical Circles (Russian Experience* by Dmitri Fomin, Sergey Genkin, and Ilia Itenberg,, and from Paul Zeitz.

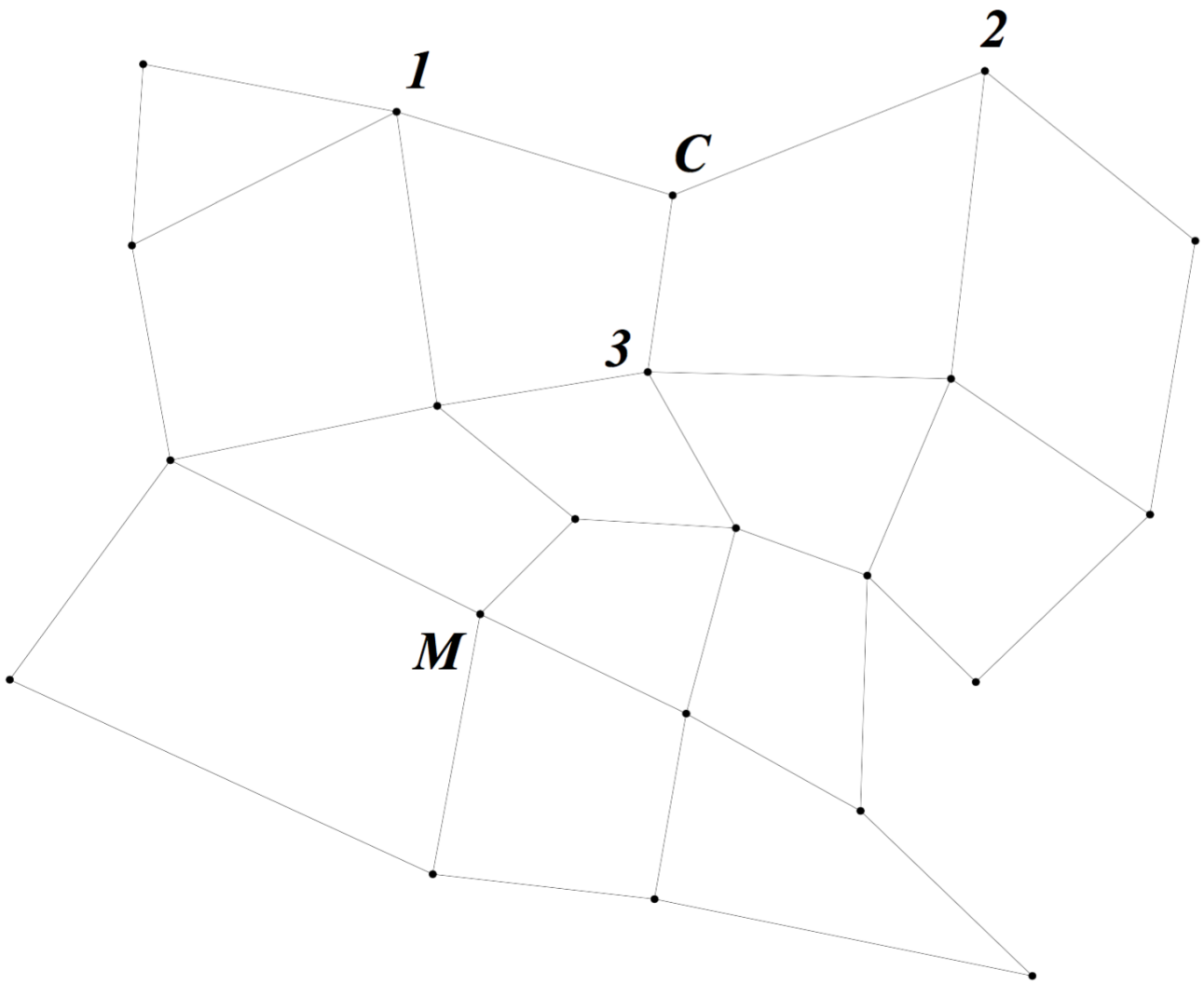
3 Evil Wizard

3. An evil wizard has imprisoned 64 math circle participants. The wizard announces, Tomorrow I will have you stand in a line, and I will put a hat on each of your heads. The hat will be colored either white or black. You will be able to see the hats of everyone in front of you, but you will not be able to see your hat or the hats of the people behind you. I will begin by asking the person at the back of the line to guess his or her hat color. If the guess is correct, that person will get a cookie. If the guess is wrong, that person will be killed in a painful way. Then I will ask the next person in line, and so on. You are only allowed to say the single word black or white when it is your turn to speak, and otherwise you are not allowed to communicate with each other while you are standing in line. Although you will not be able to see the people behind you, you will know (by hearing) if they have answered correctly or not.

   The prisoners are allowed to chat for a few minutes before their ordeal begins. What is the largest number of prisoners that can be guaranteed to survive?

4 Cat and Mouse

4. A very polite cat chases an equally polite mouse. They take turns moving on the grid depicted below.



Initially, the cat is at the point labeled C; the mouse is at M. The cat goes first, and can move to any neighboring point connected to it by a single edge. Thus the cat can go to points 1, 2, or 3, but no others, on its first turn. The cat wins if it can reach the mouse in 15 or fewer moves. Can the cat win?

5 Challenge Problems

5. The integers from 1 to 18 are written on the board in a row, in any order. Can you insert plus and minus signs between them in such a way as to get an expression that is equal to zero?

6. Twenty-five boys and 25 girls are seated at a round table. Show that both neighbors of at least one student are boys.