Hyperbolic Soccerballs

1 Building a hyperbolic soccerball.

1. Use the templates to tape together one black heptagon and two white hexagons at each vertex.

   Hints:

   - Do as little cutting as possible. Don't separate all the polygons. Instead cut around the outside of a patch of hexagons, and separate only when necessary to insert a heptagon or more hexagons.
   - Tape together along edges only. Avoid taping over vertices, where the paper won't lie flat.
   - It's easier to work with a partner.

2. How are the shapes used different from the shapes of a regular soccerball?

   A regular soccerball is said to have *positive curvature* and is a model for a geometry called spherical geometry. The hyperbolic shape is said to have *negative curvature* and is a model for hyperbolic gemetry. Regular plane geometry is called *Euclidean geometry.* What would you say is the curvature for Euclidean geometry?

2 Parallel lines

3. What is a parallel line? In Euclidean geometry, does every line have lines parallel to it? How many? How can you construct them?

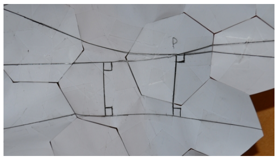
4. What about for hyperbolic geometry?

   (a) To draw a line on the model, flatten part of the model to start the line, and use a short (<15 cm) straightedge to continue the line across the model, flattening pairs of polygons as needed. Try to avoid running the line through a vertex.

   (b) After drawing your line completely across the model, you can pick it up, straighten it along the line, and sight down the line to see that it is straight.

   (c) After drawing a first line, pick a point on it and draw a short line segment m perpendicular to it. Then start a new line perpendicular to m and extend this third line across the model. What do you notice about your original line and this new, parallel line?

   (d) On one of the parallel lines from the previous step, choose a point P not lying on the common perpendicular m. Dropping a perpendicular from P to the other line, and then taking a perpendicular to that through P gives a second line through P that is parallel to the original line.



3 Triangles

5. Now, try to draw a triangle. For this it is best to try to make a big triangle.

6. Measure its interior angles.

    - One method is to mark off an arc on a small sector of a circle (cut out of a scrap of paper), lay the semicircle on a flat surface, and then use your protractor
    - It is possible to find the sum of the three angles by marking off three consecutive arcs along a small sector of a circle.

7. What is the sum of the interior angles of the triangle?

8. How far does the sum deviate from 180°?

9. Compare the deviation of the sum from 180° to the number vertices inside the triangle. The vertices are where three shapes meet.

10. Draw a triangle that encloses as many vertices as possible.

4 Angle Defect

11. If you tile the plane with squares, four squares meet at each vertex, and each square has an angle of 90°, so the sum of all the angles is 360° at every vertex. What if you tile the plane with hexagons instead? What is the sum of the angles at each vertex?

12. What is the sum of angles at one vertex of your hyperbolic soccerball? The deviation of the sum of the angles from 360° is called the angle defect.

13. The total angle defect inside a region is the sum of all the angle defects at all vertices inside the region. What is the total angle defect inside one of your triangles?

14. Try to relate the total angle defect to the sum of the angles of the triangle. This relationship is a special case of the Gauss-Bonnet Theorem.

15. Does this same relationship hold for quadrilaterals instead of triangles?