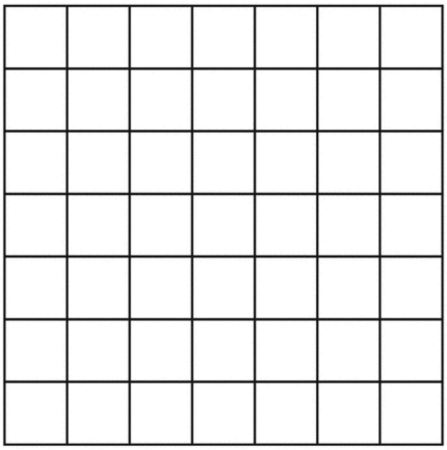
Grid Power (a.k.a. Counting Squares) [1]

Consider a 7 x 7 square of graph paper.



1. How many 1 x 1 squares are there on this 7 x 7 square?
2. How many 2 x 2 squares?
3. How many squares of any size?

   Now, say that the sides of the squares may or may not be parallel to the grid lines.

4. How many squares with sides less than 2 can you find now?
5. What about squares of any size? Try to find a systematic way to count them.
6. For grids of other sizes (e.g. 6 x 6, or 5 x 5, or 8 x 8, or 9 x 9), count the total number of squares, the number of "tilted" squares, and the number "non-tilted" squares. What do you notice about these numbers?

[1] Problems from Tatiana Shubin, Joshua Zucker, and the Julia Robinson Math Festival. See Tatiana Shubin's YouTube video series for more.